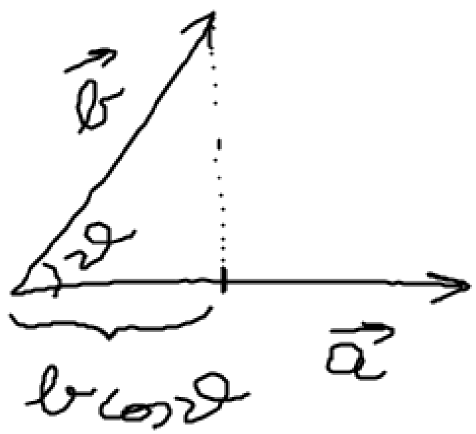
PRODOTTO SCALARE

$\theta \quad \vartheta$

$$\vec{a} \cdot \vec{b} = a b \cos\vartheta$$

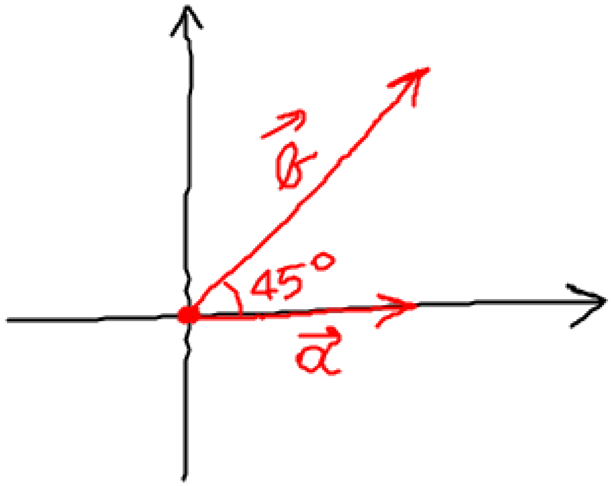


$$\vec{a} = (a_x, a_y)$$

$$\vec{b} = (b_x, b_y)$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$

ESEMPIO



$|\vec{b}| = 3$

$\vec{a} = (2, 0) \quad |\vec{a}| = 2$

$\vec{b} = \left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

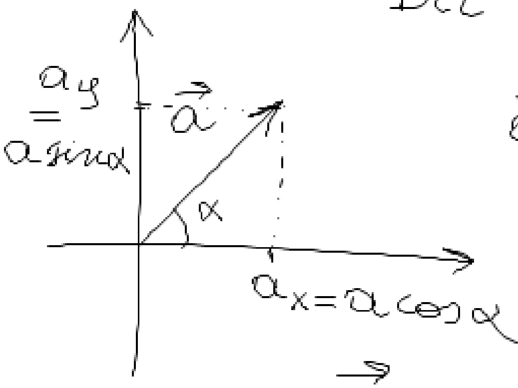
1° MODO

$$\vec{a} \cdot \vec{b} = a\, b \cos 45° =$$
$$= 2 \cdot 3 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$$

2° MODO

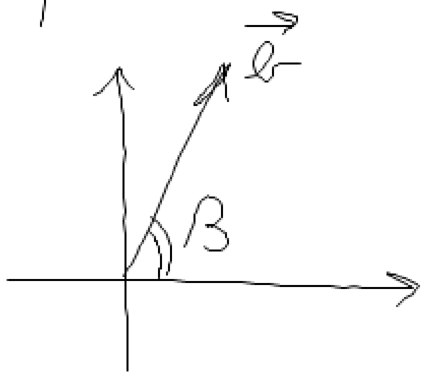
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y =$$
$$= 2 \cdot \frac{3\sqrt{2}}{2} + 0 \cdot \frac{3\sqrt{2}}{2} = 3\sqrt{2}$$

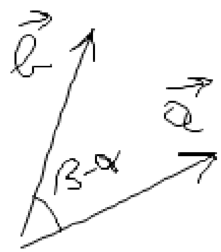
DIMOSTRAZIONE DELLA PROPRIETÀ DEL PRODOTTO SCALARE



$$\vec{a} = (a_x, a_y) = (a\cos\alpha, a\sin\alpha)$$

$$\vec{b} = (b_x, b_y) = (b\cos\beta, b\sin\beta)$$





$$\vec{a}\cdot\vec{b} = a\,b\,\cos(\beta-\alpha)$$

FORMULA DI SOTTRAZIONE $\longrightarrow$ $\cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$

$$\vec{a}\cdot\vec{b} = ab\cos(\beta-\alpha) =$$

$$= ab\cos\beta\cos\alpha + ab\sin\beta\sin\alpha =$$

$$= \underbrace{(a\cos\alpha)}_{a_x}\underbrace{(b\cos\beta)}_{b_x} + \underbrace{(a\sin\alpha)}_{a_y}\underbrace{(b\sin\beta)}_{b_y} =$$

$$= a_x b_x + a_y b_y$$