321 Una funzione $f(x)$ ha le seguenti proprietà:
i) $f(1) = 1$; ii) $f(2x) = 4f(x) + 6$; iii) $f(x+2) = f(x) + 12x + 12$. Calcola $f(6)$.

(CAN *Canadian Open Mathematics Challenge*, 2004)
[106]

$f(1) = 1$ $\quad f(2x) = 4f(x) + 6$

$f(x+2) = f(x) + 12x + 12$

$f(6) = f(4+2) = f(4) + 12 \cdot 4 + 12 =$

$= f(2 \cdot 2) + 48 + 12 =$

$= 4 \cdot f(2) + 6 + 60 =$

$= 4 \cdot f(2 \cdot 1) + 66 = 4[4 \cdot f(1) + 6] + 66 =$ ($f(1) = 1$)

$= 4[4 \cdot 1 + 6] + 66 = 40 + 66 = 106$

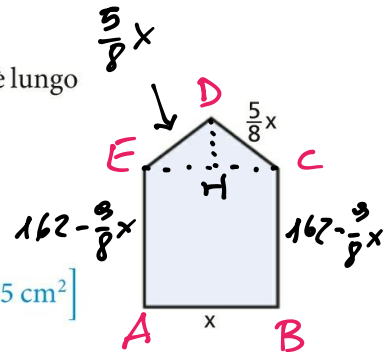
323 La finestra di un'antica chiesa ha la forma rappresentata nella figura; il suo perimetro è lungo 324 cm.

a. Scrivi la misura dell'area A in funzione della base x della finestra.

b. Scrivi il dominio della funzione A.

c. Se la base è lunga 1 m, quanto vale l'area?

$\left[A(x) = 162x - \frac{15}{16}x^2;\ D:]0; 144[;\ 6825\ \text{cm}^2\right]$



$\overline{EA} = \overline{CB} = ?$

$2p = 2\overline{EA} + x + 2 \cdot \frac{5}{8}x$ $\Rightarrow$ POSSO RICAVARE $\overline{EA}$ IN FUNZIONE DI x

PERIMETRO = 324

$324 = 2\overline{EA} + x + \frac{5}{4}x$

$2\overline{EA} = 324 - \frac{9}{4}x \Rightarrow \overline{EA} = 162 - \frac{9}{8}x$

$\overline{DH}^2 = \overline{DC}^2 - \overline{HC}^2 = \frac{25}{64}x^2 - \frac{x^2}{4} = \frac{9}{64}x^2 \Rightarrow \overline{DH} = \frac{3}{8}x$

$A(x) = x\left(162 - \frac{9}{8}x\right) + \frac{1}{2}x \cdot \frac{3}{8}x = 162x - \frac{9}{8}x^2 + \frac{3}{16}x^2 =$

$= 162x - \frac{15}{16}x^2$

↑ AREA — AREA ABCE — AREA CDE

DOMINIO $162x - \frac{15}{16}x^2 > 0 \Rightarrow \frac{15}{16}x^2 - 162x < 0$

$x = 0$

$x\left(\frac{15}{16}x - 162\right) < 0$

$\frac{15}{16}x - 162 = 0 \Rightarrow 15x = 162 \cdot 16$

$x = \frac{162 \cdot 16}{15}$

SICURAMENTE DEVE ESSERE $0 < x < \frac{162 \cdot 16}{15}$

MA PER DARE SENSO A $\overline{CB}$

$162 - \frac{9}{8}x > 0 \Rightarrow -\frac{9}{8}x > -162$

$x < \frac{8 \cdot 162}{9}$ ^{18} $= 144$

DOMINIO $\Rightarrow$ $0 < x < 144$ $\rightarrow$ $(0, 144)$ oppure $]0, 144[$

L'area è in cm^2.

$$\mathcal{A}(x) = 162x - \frac{15}{16}x^2$$

$x = 1\,m = 100\,cm$

$$\mathcal{A}(100) = 162 \cdot 100 - \frac{15}{16}10000 = 6825 \ (cm^2)$$

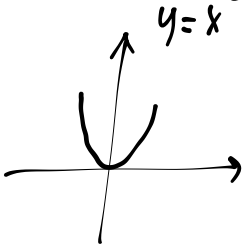
396) $y = -x^2 + 6|x|$

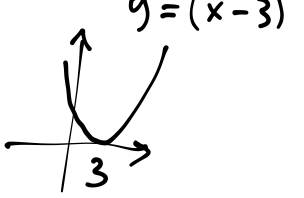
$y = -x^2 + 6x$

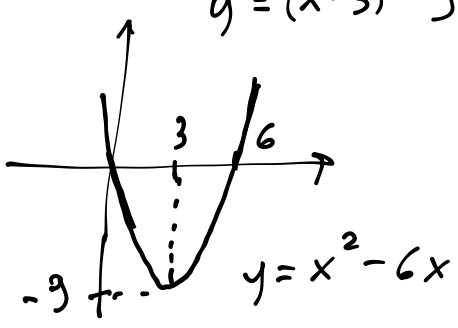
parte ob

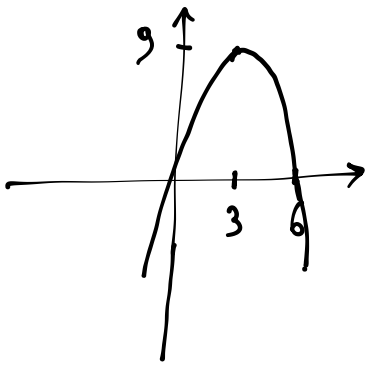
$y = x^2 - 6x = x^2 - 6x + 9 - 9 =$

$= (x-3)^2 - 9$

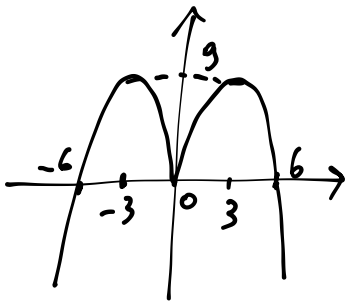








$y = -x^2 + 6x \xrightarrow{A(|x|)} y = -|x|^2 + 6|x|$



$y = -x^2 + 6|x|$