28/11/2017 ALCUNI ESERCIZI DELLA VERIFICA

comporre

$$f(x) = \begin{cases} 2x^2 - 1 & \text{se } x \leq 0 \\ -2x+5 & \text{se } x > 0 \end{cases} \qquad g(x) = x^2 + 1$$

APPLICO SOLO LA PARTE DI f RELATIVA A INPUT POSITIVI

$$(f \circ g)(x) = f(g(x)) = f(\underbrace{x^2+1}_{\text{sempre } > 0}) = -2(x^2+1) + 5 =$$

$$= -2x^2 - 2 + 5 = \boxed{-2x^2 + 3}$$

$$(g \circ f)(x) = g(f(x)) = \begin{cases} g(2x^2-1) & \text{se } x \leq 0 \\ g(-2x+5) & \text{se } x > 0 \end{cases} = \begin{cases} (2x^2-1)^2 + 1 & \text{se } x \leq 0 \\ (-2x+5)^2 + 1 & \text{se } x > 0 \end{cases}$$

$$= \begin{cases} 4x^4 + 1 - 4x^2 + 1 & \text{se } x \leq 0 \\ 4x^2 + 25 - 20x + 1 & \text{se } x > 0 \end{cases}$$

$$= \begin{cases} 4x^4 - 4x^2 + 2 & \text{se } x \leq 0 \\ 4x^2 - 20x + 26 & \text{se } x > 0 \end{cases}$$

2) INVERTIRE

$g : \mathbb{R} - \{3\} \to \mathbb{R} - \{2\}$ $\quad g(x) = \dfrac{2x-1}{x-3} \rightsquigarrow y = \dfrac{2x-1}{x-3}$

$$x = \frac{2y-1}{y-3}$$

$$xy - 3x = 2y - 1$$

$$xy - 2y = 3x - 1$$

$$y = \frac{3x-1}{x-2} \longleftarrow y(x-2) = 3x - 1$$

$$g^{-1}(x) = \frac{3x-1}{x-2}$$

3) STABILIRE SE PARI O DISPARI

$$f(x) = 2x^4 - x^3$$

$$f(-x) = 2(-x)^4 - (-x)^3 = 2x^4 + x^3$$

$$-f(x) = -2x^4 + x^3$$

$\cancel{\forall x \quad f(x) = f(-x)}$ $\qquad$ $\cancel{\forall x \quad f(-x) = -f(x)}$

ad es. $x = 1 \qquad f(1) = 1 \qquad f(-1) = 2 + 1 = 3$

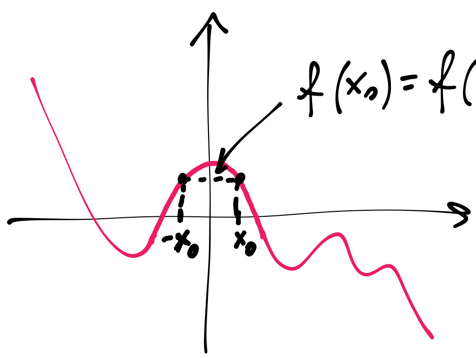
$f(1) \neq f(-1)$ NÉ PARI

$[f(1) \neq -f(-1)]$ NÉ DISPARI

$f(-1) \neq -f(1)$

$$g(x) = \frac{2x^3}{1-|x|}$$ è DISPARI

$$g(-x) = \frac{2(-x)^3}{1-|-x|} = -\frac{2x^3}{1-|x|} = -g(x)$$



$f(x_0) = f(-x_0)$ però non è pari perché deve valere $\forall x$

4) $$(2k+1)x-(3k+2)y-5k-2=0$$

$$2kx+x-3ky-2y-5k-2=0$$

$$x-2y-2+k(2x-3y-5)=0$$

1° generatrice

$x-2y-2=0$

2° generatrice

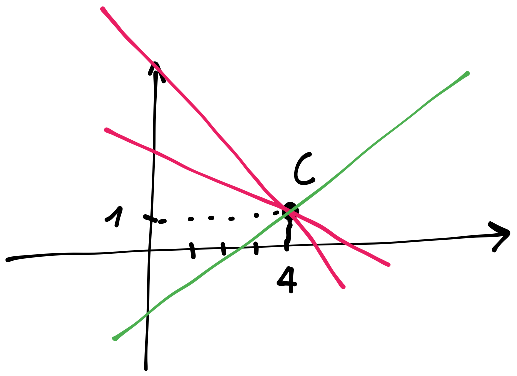
$2x-3y-5=0$ (esclusa)

$C(4,1)$

rette del fascio // agli assi

$x=4 \qquad y=1$

f)



Coeff. angolare (generico)

$$m=\frac{2k+1}{3k+2}$$

$$\frac{2k+1}{3k+2}<0$$

N] $2k+1>0 \rightarrow k>-\frac{1}{2}$

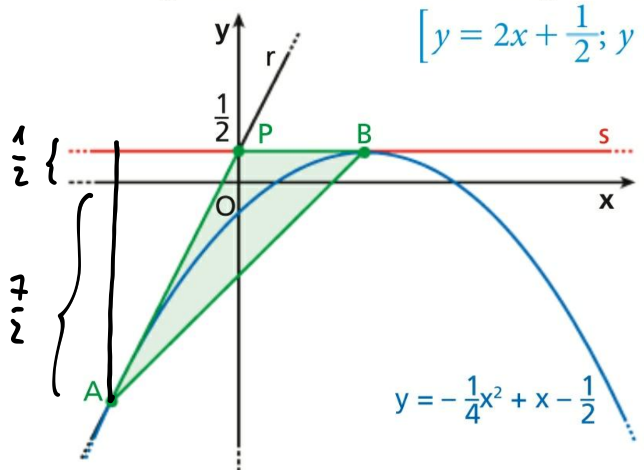
D] $3k+2>0 \rightarrow k>-\frac{2}{3}$

	$-\frac{2}{3}$		$-\frac{1}{2}$	
$-$		$-$		$+$
$-$		$+$		$+$
$+$		$-$		$+$

$$\boxed{-\frac{2}{3}<k<-\frac{1}{2}}$$

299 Scrivi le equazioni delle rette r e s passanti per P e tangenti alla parabola utilizzando le informazioni della figura; calcola l'area del triangolo ABP.

$\left[y = 2x + \frac{1}{2}; y = \frac{1}{2}; 4\right]$



$s: \ y = \frac{1}{2}$

$P\left(0, \frac{1}{2}\right)$

$y - \frac{1}{2} = m(x - 0)$

$y = mx + \frac{1}{2}$ retta per P

$\begin{cases} y = -\frac{1}{4}x^2 + x - \frac{1}{2} \\ y = mx + \frac{1}{2} \end{cases}$

$mx + \frac{1}{2} = -\frac{1}{4}x^2 + x - \frac{1}{2}$

$\frac{1}{4}x^2 + mx - x + 1 = 0$

$\frac{1}{4}x^2 + (m - 1)x + 1 = 0$

$\Delta = 0 \qquad (m-1)^2 - 4 \cdot \frac{1}{4} \cdot 1 = 0 \qquad m^2 + \cancel{1} - 2m - \cancel{1} = 0$

$m^2 - 2m = 0$

$m(m - 2) = 0 \quad m = 0 \ \vee \ m = 2$

$m = 0 \Rightarrow s: \ y = \frac{1}{2}$

$m = 2 \Rightarrow r: \ y = 2x + \frac{1}{2}$

$B \equiv V$ (vertice) $\qquad y = -\frac{1}{4}x^2 + x - \frac{1}{2}$

$x_B = -\frac{b}{2a} = -\frac{1}{-\frac{1}{2}} = 2 \qquad y_B = -1 + 2 - \frac{1}{2} = \frac{1}{2}$

NON SERVIVA....

$B\left(2, \frac{1}{2}\right)$

$A \begin{cases} y = 2x + \frac{1}{2} \\ y = -\frac{1}{4}x^2 + x - \frac{1}{2} \end{cases}$

$2x + \frac{1}{2} = -\frac{1}{4}x^2 + x - \frac{1}{2}$

$\frac{1}{4}x^2 + x + 1 = 0 \qquad \Delta = 0$

$x = \frac{-1}{\frac{1}{2}} = -2 \rightsquigarrow y = -4 + \frac{1}{2} = -\frac{7}{2}$

$A\left(-2, -\frac{7}{2}\right)$

Area $P(0,\frac{1}{2})$ $A(-2,-\frac{7}{2})$ $B(2,\frac{1}{2})$

BASE $\overline{PB} = 2$ ALTEZZA RELATIVA A PB È $\frac{7}{2}+\frac{1}{2}=4$

$$\text{Area} = \frac{1}{2}\cdot 2\cdot 4 = \boxed{4}$$

305 Trova l'equazione della tangente comune alle due parabole di equazioni

$y = -x^2 - 2x$ e $y = -x^2 + 2x + 3$.

$\left[y = \frac{3}{2}x + \frac{49}{16}\right]$

$y = mx + q$ INCOGNITE m, q

$$\begin{cases} y = -x^2 - 2x \\ y = mx + q \end{cases}$$

$mx + q = -x^2 - 2x$

$x^2 + mx + 2x + q = 0$ $x^2 + (m+2)x + q = 0$

$\Delta = 0 \Rightarrow \boxed{(m+2)^2 - 4q = 0}$

$$\begin{cases} y = -x^2 + 2x + 3 \\ y = mx + q \end{cases}$$

$mx + q = -x^2 + 2x + 3$ $x^2 + (m-2)x + q - 3 = 0$

$x^2 + mx - 2x + q - 3 = 0$

$\Delta = 0 \Rightarrow \boxed{(m-2)^2 - 4(q-3) = 0}$ DA METTERE A SISTEMA

$$\begin{cases} m^2 + 4 + 4m - 4q = 0 \\ -m^2 - 4 + 4m + 4q - 12 = 0 \end{cases}$$

$// \quad // \quad 8m \quad // \quad -12 = 0$ $m = \frac{12}{8} = \frac{3}{2}$

$$\begin{cases} \frac{9}{4} + 4 + 6 - 4q = 0 \\ m = \frac{3}{2} \end{cases}$$

$$\begin{cases} 4q = \frac{49}{4} \Rightarrow q = \frac{49}{16} \\ m = \frac{3}{2} \end{cases}$$

$\boxed{y = \frac{3}{2}x + \frac{49}{16}}$ TANGENTE COMUNE

325 $V(2;1)$, $F\left(2;\frac{3}{4}\right)$.

$[y=-x^2+4x-3]$

Trovare l'eq. della parabola

$$y=ax^2+bx+c$$

$$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) \qquad F\left(-\frac{b}{2a}, \frac{1-\Delta}{4a}\right)$$

$$V(2,1) \qquad F\left(2, \frac{3}{4}\right)$$

$$\begin{cases} -\frac{b}{2a}=2 \\ -\frac{\Delta}{4a}=1 \\ \frac{1-\Delta}{4a}=\frac{3}{4} \end{cases} \quad \begin{cases} b=-4a \\ \Delta=-4a \\ 1-\Delta=3a \end{cases} \quad \begin{cases} b=-4a \\ \Delta=-4a \\ 1+4a=3a \end{cases} \quad \begin{cases} b=4 \\ \Delta=4 \\ a=-1 \end{cases}$$

$$\Delta=b^2-4ac \Rightarrow \quad 4=16+4c \Rightarrow 4c=-12 \Rightarrow c=-3$$

$$y=ax^2+bx+c \qquad \boxed{y=-x^2+4x-3}$$

352 $A(1;0)$, $B(0;-5)$, $C(2;3)$.

PARABOLA $y=ax^2+bx+c$

Pass. per $A \rightarrow$, $B \rightarrow$, $C \rightarrow$

$$\begin{cases} 0=a+b+c \\ -5=0+0+c \\ 3=4a+2b+c \end{cases} \quad \begin{cases} a+b+c=0 \\ c=-5 \\ 4a+2b+c=3 \end{cases}$$

$$\begin{cases} a+b-5=0 \\ // \\ 4a+2b-5=3 \end{cases} \quad \begin{cases} a+b=5 \\ 4a+2b=8 \end{cases} \quad \begin{cases} a=5-b \\ 20-4b+2b=8 \end{cases}$$

$$\begin{cases} \\ -2b=-12 \end{cases} \quad \begin{cases} a=-1 \\ c=-5 \\ b=6 \end{cases} \quad \boxed{y=-x^2+6x-5}$$