5/12/2017

379 Determina l'equazione della parabola $y = ax^2 + bx + c$ passante per il punto $A(0;\ 1)$ e tangente a entrambe le rette di equazioni $y = -4x$ e $4x + 4y - 3 = 0$.

$[y = x^2 - 2x + 1;\ y = 9x^2 + 2x + 1]$

$A(0,1) \rightsquigarrow 1 = c \qquad y = ax^2 + bx + 1$

$\begin{cases} y = -4x \\ y = ax^2 + bx + 1 \end{cases} \Rightarrow ax^2 + bx + 1 = -4x$

$ax^2 + bx + 4x + 1 = 0$

$ax^2 + (b+4)x + 1 = 0$

$\Delta = 0 \qquad \boxed{(b+4)^2 - 4a = 0}$

$\begin{cases} 4x + 4y - 3 = 0 \Rightarrow y = -x + \frac{3}{4} \\ y = ax^2 + bx + 1 \end{cases} \qquad ax^2 + bx + 1 = -x + \frac{3}{4}$

$ax^2 + (b+1)x + \frac{1}{4} = 0$

$\Delta = 0 \qquad \boxed{(b+1)^2 - a = 0}$

$\begin{cases} b^2 + 16 + 8b - 4a = 0 \\ b^2 + 1 + 2b - a = 0 \end{cases}$

$// \quad 15 + 6b - 3a = 0$

$\begin{cases} 15 + 6b - 3a = 0 \\ b^2 + 2b + 1 - a = 0 \end{cases}$

$\hookrightarrow a = b^2 + 2b + 1$

$\begin{cases} 15 + 6b - 3(b^2 + 2b + 1) = 0 \\ a = b^2 + 2b + 1 \end{cases} \qquad \begin{cases} 15 + \cancel{6b} - 3b^2 - \cancel{6b} - 3 = 0 \\ // \end{cases}$

$\begin{cases} b^2 = 4 \\ \end{cases} \begin{matrix} \nearrow \begin{cases} b = 2 \\ a = 4 + 4 + 1 = 9 \end{cases} \\ \searrow \begin{cases} b = -2 \\ a = 4 - 4 + 1 = 1 \end{cases} \end{matrix}$

$\boxed{\begin{matrix} y = 9x^2 + 2x + 1 \\ \vee \\ y = x^2 - 2x + 1 \end{matrix}}$

389 Scrivi l'equazione della parabola che ha per direttrice la retta di equazione $y = -\frac{11}{2}$ e il fuoco in $F\left(-4; -\frac{9}{2}\right)$, e determina l'equazione della retta tangente passante per il punto A della parabola di ascissa -6. $\left[y = \frac{1}{2}x^2 + 4x + 3;\ y = -2x - 15\right]$

$y = ax^2 + bx + c$

asse // asse y

$F\left(-\frac{b}{2a}, \frac{1-\Delta}{4a}\right)$ $\quad d:\ y = -\frac{1+\Delta}{4a}$

$$\begin{cases} -\frac{b}{2a} = -4 \\ \frac{1-\Delta}{\cancel{4}_2 a} = -\frac{9}{\cancel{2}_1} \\ -\frac{1+\Delta}{4a} = -\frac{11}{2} \end{cases} \quad \begin{cases} b = 8a \\ 1-\Delta = -18a \\ 1+\Delta = 22a \end{cases}$$

$2 \; // = 4a$

$$\begin{cases} b = 4 \\ \Delta = 22a - 1 = 10 \\ a = \frac{1}{2} \end{cases}$$

$\Delta = b^2 - 4ac \qquad 10 = 16 - 2c \qquad 2c = 6 \qquad c = 3$

$$\boxed{y = \frac{1}{2}x^2 + 4x + 3}$$

$A(-6, ?) \quad y_A = \frac{1}{2}(-6)^2 + 4(-6) + 3 = 18 - 24 + 3 = -3$

$A(-6, -3)$

$y + 3 = m(x+6)$

$\downarrow$

$y + 3 = mx + 6m$

$y = mx + 6m - 3$

$$\begin{cases} y = \frac{1}{2}x^2 + 4x + 3 \\ y = mx + 6m - 3 \end{cases}$$

$\frac{1}{2}x^2 + 4x + 3 = mx + 6m - 3$

$\frac{1}{2}x^2 + 4x - mx + 6 - 6m = 0$

$\frac{1}{2}x^2 + (4-m)x + 6 - 6m = 0$

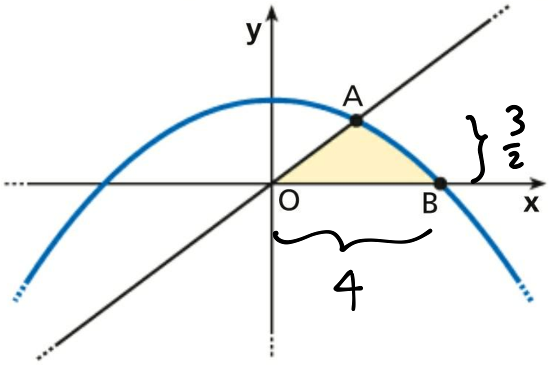
$\Delta = 0 \quad (4-m)^2 - 4 \cdot \frac{1}{2}(6-6m) = 0$

$16 + m^2 - 8m - 12 + 12m = 0$

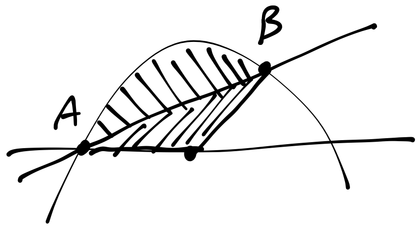
$(m+2)^2 = 0 \qquad m = -2$

$$\boxed{y = -2x - 15}$$

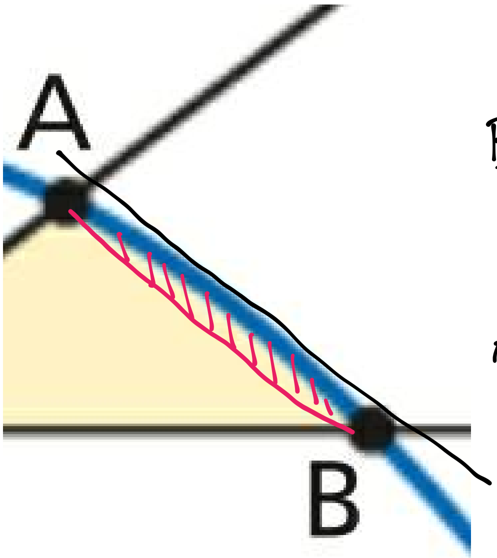
320 Trova l'area del triangolo mistilineo OAB rappresentato nella figura, sapendo che la parabola ha equazione $y = -\frac{1}{8}x^2 + 2$ e la retta $y = \frac{3}{4}x$.



$\left[\frac{19}{6}\right]$



Trovo A e B



$$B\begin{cases} y = -\frac{1}{8}x^2 + 2 \\ y = 0 \end{cases} \quad -\frac{1}{8}x^2 + 2 = 0 \Rightarrow x = \pm 4 \quad x = 4 \quad B(4, 0)$$

$$A\begin{cases} y = -\frac{1}{8}x^2 + 2 \\ y = \frac{3}{4}x \end{cases} \quad -\frac{1}{8}x^2 + 2 = \frac{3}{4}x \quad -x^2 + 16 = 6x \quad x^2 + 6x - 16 = 0$$

$$x = -3 + \sqrt{25} = -3 + 5 = 2 \qquad A\left(2, \frac{3}{2}\right)$$

Per trovare l'area del segmento parabolico mi serve la tangente alla parabola parallela alla retta AB

$$m_{AB} = \frac{\frac{3}{2} - 0}{2 - 4} = \frac{\frac{3}{2}}{-2} = -\frac{3}{4}$$

$$\begin{cases} y = -\frac{3}{4}x + K \leftarrow \text{retta // ad } AB \\ y = -\frac{1}{8}x^2 + 2 \end{cases} \quad -\frac{1}{8}x^2 + 2 = -\frac{3}{4}x + K$$

$$\frac{1}{8}x^2 - \frac{3}{4}x + K - 2 = 0$$

$$\Delta = 0$$

$$\frac{9}{\cancel{16}_{\,2}} - 4 \cdot \frac{1}{\cancel{8}}(K - 2) = 0$$

$$\frac{9}{2} - 4K + 8 = 0 \qquad 9 - 8K + 16 = 0$$

$$-8K = -25 \qquad K = \frac{25}{8} \Rightarrow y = -\frac{3}{4}x + \frac{25}{8}$$

$$8y = -6x + 25$$

TANGENTE $\boxed{6x + 8y - 25 = 0}$

CALCOLO LA DISTANZA TRA LA RETTA AB E LA TANGENTE

USO IL PUNTO B ⇝ CALCOLO LA DISTANZA DI B DALLA TANGENTE

$B(4,0)$ $\qquad 6x+8y-25=0$

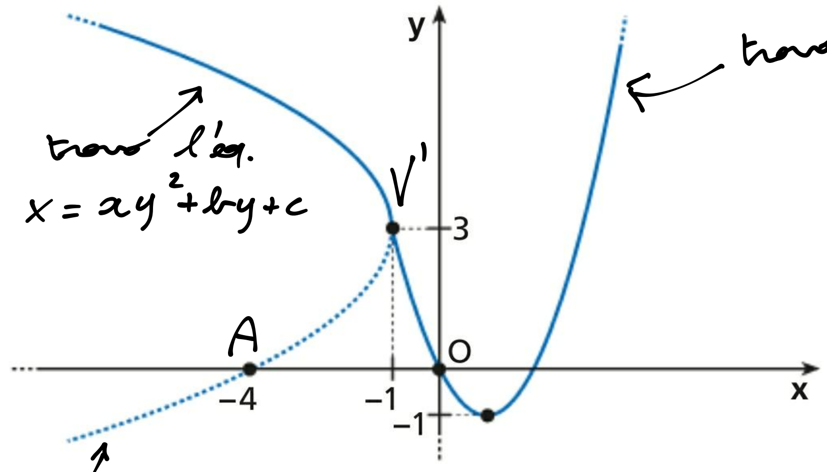
$$d=\frac{|6\cdot 4+8\cdot 0-25|}{\sqrt{6^2+8^2}}=\frac{1}{\sqrt{36+64}}=\frac{1}{10}$$

$A\left(2,\frac{3}{2}\right)$ $\qquad \overline{AB}=\sqrt{(2-4)^2+\left(\frac{3}{2}-0\right)^2}=\sqrt{4+\frac{9}{4}}=\sqrt{\frac{25}{4}}=\frac{5}{2}$

$$\mathcal{A}_{\text{SEGMENTO}}=\frac{2}{3}\cdot\frac{5}{2}\cdot\frac{1}{10}=\frac{1}{6} \qquad \mathcal{A}_{\text{TRIANGOLO}}=\frac{1}{2}\cdot 4\cdot\frac{3}{2}=3$$

$$\mathcal{A}_{\text{REGIONE}}=\frac{1}{6}+3=\boxed{\frac{19}{6}}$$

427



trovo l'eq. $x = ay^2 + by + c$

trovo l'equazione $y = ax^2 + bx + c$

$V'(-1, 3)$

$O(0, 0)$

$-\dfrac{\Delta}{4a} = -1$

$$\left[y = \begin{cases} 3 + \sqrt{-3x-3} & \text{se } x \le -1 \\ x^2 - 2x & \text{se } x > -1 \end{cases} \right]$$

$$\begin{array}{l} V' \\ O \\ \\ \end{array} \begin{cases} 3 = a - b + c \\ c = 0 \\ \Delta = 4a \end{cases}$$

$b^2 - 4ac = 4a$

$b^2 = 4a \quad a > 0$

le parte tratteggiata non fa parte del grafico

$x = ay^2 + by + c$

$V(-1, 3) \quad A(-4, 0)$

$$\begin{array}{l} \\ V \\ A \end{array} \begin{cases} -\dfrac{b}{2a} = 3 \\ -1 = 9a + 3b + c \\ -4 = c \end{cases}$$

$$\begin{cases} b = -6a \\ 9a - 18a - 4 = -1 \\ c = -4 \end{cases} \qquad \begin{cases} b = -6a \\ -9a = 3 \\ c = -4 \end{cases}$$

$$\begin{cases} a = -\dfrac{1}{3} \\ b = 2 \\ c = -4 \end{cases} \qquad \boxed{x = -\frac{1}{3} y^2 + 2y - 4}$$

$$\begin{cases} 3 = a - b \\ b^2 = 4a \\ c = 0 \end{cases} \qquad \begin{cases} b = a - 3 \\ (a-3)^2 = 4a \\ c = 0 \end{cases}$$

$$\begin{cases} b = a - 3 \\ a^2 + 9 - 6a - 4a = 0 \\ c = 0 \end{cases} \qquad \begin{cases} b = a - 3 \\ a^2 - 10a + 9 = 0 \\ c = 0 \end{cases}$$

$$\begin{cases} b = a - 3 \\ (a-1)(a-9) = 0 \\ c = 0 \end{cases}$$

$$\boxed{\begin{cases} a = 1 \\ b = -2 \\ c = 0 \end{cases}}$$

scelgo perché il vertice ha ascisse > 0

$$\begin{cases} a = 9 \\ b = 6 \\ c = 0 \end{cases}$$

vertice con ascisse < 0

$$\boxed{y = x^2 - 2x}$$

$$\boxed{x=-\frac{1}{3}y^2+2y-4}$$

$$\underbrace{\frac{1}{3}}_{a}y^2\underbrace{-2}_{b}y+\underbrace{4+x}_{c}=0$$

$$\Delta = 4-4\cdot\frac{1}{3}(4+x) = 4-\frac{16}{3}-\frac{4}{3}x =$$

$$=-\frac{4}{3}-\frac{4}{3}x \geq 0$$

$$\hookrightarrow \boxed{x\leq -1}$$

$$y=\frac{2\pm\sqrt{-\frac{4}{3}-\frac{4}{3}x}}{\frac{2}{3}}$$

SCELGO IL RAMO CORRISPONDENTE A +

$$y=\frac{2+\sqrt{\frac{-4-4x}{3}}}{\frac{2}{3}}=\frac{2+2\sqrt{\frac{-1-x}{3}}}{\frac{2}{3}}=$$

$$=\frac{\cancel{2}\left(1+\sqrt{\frac{-1-x}{3}}\right)}{\cancel{2}}\cdot 3 = 3+3\sqrt{\frac{-1-x}{3}}=$$

$$=3+\sqrt{\frac{3^{\cancel{2}}(-1-x)}{\cancel{3}}}=3+\sqrt{-3-3x}$$

$$y=\begin{cases}3+\sqrt{-3x-3} & \text{se } x\leq -1\\ x^2-2x & \text{se } x\geq -1\end{cases}$$