10/10/2018

38 $$\frac{2}{\csc(90^\circ-\alpha)}+6\frac{\cos(180^\circ-\alpha)}{\sin(-\alpha)}-2\cos(180^\circ-\alpha) =$$

$$= 2\sin(90^\circ-\alpha)+6\,\frac{-\cos\alpha}{-\sin\alpha}+2\cos\alpha =$$

$$= 2\cos\alpha+6\cot\alpha+2\cos\alpha = 4\cos\alpha+6\cot\alpha$$

39 $$\sin\Big(\overset{\alpha+2\pi-\frac{\pi}{2}}{\alpha+\frac{3}{2}\pi}\Big)\cos(\alpha+\pi)-\frac{\tan\Big(\overset{\pi+\frac{\pi}{2}}{\frac{3}{2}\pi-\alpha}\Big)\sin\left(\frac{\pi}{2}+\alpha\right)}{\sin(-\alpha)+\cos\left(\frac{\pi}{2}+\alpha\right)} =$$

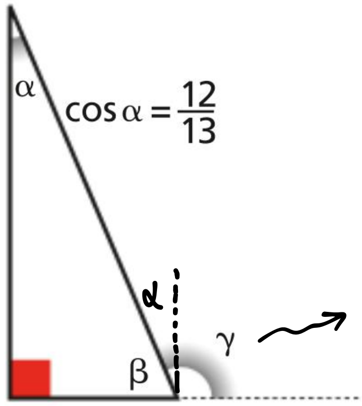
$$= \underbrace{\sin\left(\alpha-\frac{\pi}{2}\right)}_{-\sin\left(\frac{\pi}{2}-\alpha\right)}(-\cos\alpha)-\frac{\tan\left(\frac{\pi}{2}-\alpha\right)\cos\alpha}{-\sin\alpha-\sin\alpha} =$$

$$= -\cos\alpha(-\cos\alpha)-\frac{\cot\alpha\cdot\cos\alpha}{-2\sin\alpha} =$$

$$= \cos^2\alpha+\frac{\frac{\cos^2\alpha}{\sin\alpha}}{2\sin\alpha} = \cos^2\alpha+\frac{1}{2}\,\frac{\cos^2\alpha}{\sin^2\alpha} =$$

$$= \cos^2\alpha+\frac{1}{2}\cot^2\alpha$$

50



$\gamma = \alpha + \frac{\pi}{2}$

Calcola: $\tan\beta$, $\cos\gamma$, $\sin(\pi+\gamma)$.

$\left[\frac{12}{5}; -\frac{5}{13}; -\frac{12}{13}\right]$

$$\beta = \frac{\pi}{2} - \alpha \qquad \tan\beta = \tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha = \frac{\frac{12}{13}}{\frac{5}{13}} = \boxed{\frac{12}{5}}$$

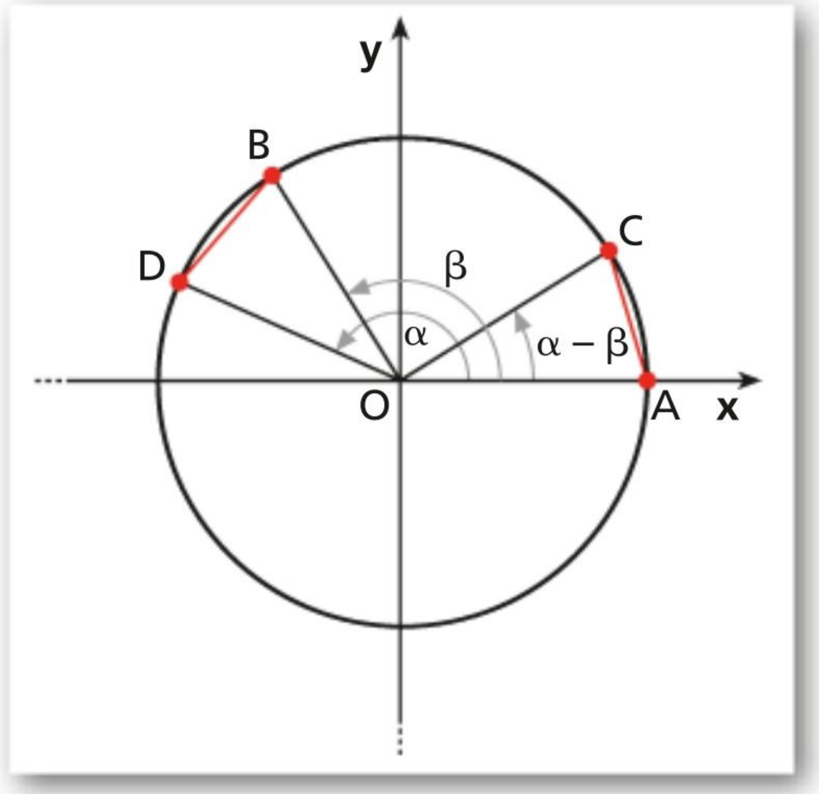
$$\sin\alpha = +\sqrt{1-\cos^2\alpha} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\cos\gamma = \cos(\pi-\beta) = -\cos\beta = -\cos\left(\frac{\pi}{2}-\alpha\right) = -\sin\alpha = \boxed{-\frac{5}{13}}$$

$$\sin(\pi+\gamma) = -\sin\gamma = -\sin(\pi-\beta) = -\sin\beta = -\sin\left(\frac{\pi}{2}-\alpha\right) =$$

$$= -\cos\alpha = \boxed{-\frac{12}{13}}$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$



$A(1,0)$ $C(\cos(\alpha-\beta), \sin(\alpha-\beta))$

$B(\cos\beta, \sin\beta)$ $D(\cos\alpha, \sin\alpha)$

$$\overline{AC}^2=\overline{BD}^2$$

$$[1-\cos(\alpha-\beta)]^2+\sin^2(\alpha-\beta)=[\cos\beta-\cos\alpha]^2+[\sin\beta-\sin\alpha]^2$$

$$1-2\cos(\alpha-\beta)+\underbrace{\cos^2(\alpha-\beta)+\sin^2(\alpha-\beta)}_{1}=$$

$$=\cos^2\beta-2\cos\alpha\cos\beta+\cos^2\alpha+\sin^2\beta-2\sin\alpha\sin\beta+\sin^2\alpha$$

$$1 \qquad 1$$

$$\cancel{2}-2\cos(\alpha-\beta)=\cancel{2}-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta$$

$$\boxed{\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta}$$

QED