11/10/2018

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos(\alpha-(-\beta)) =$$

$$= \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta) =$$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\underbrace{\alpha-\beta}_{\gamma}) = \cos\left(\frac{\pi}{2}-\gamma\right) = \cos\left(\frac{\pi}{2}-(\alpha-\beta)\right) =$$

$$= \cos\left(\frac{\pi}{2}-\alpha+\beta\right) = \cos\left(\left(\frac{\pi}{2}-\alpha\right)+\beta\right) =$$

$$= \cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta - \sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta =$$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin(\alpha-(-\beta)) = \sin\alpha\cos(-\beta) - \cos\alpha\sin(-\beta) =$$

$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Funzione	Formula di addizione	Formula di sottrazione
seno	$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
coseno	$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} =$$

$$= \frac{\underbrace{\frac{\sin\alpha}{\cos\alpha}}_{\tan\alpha}\frac{\cancel{\cos\beta}}{\cancel{\cos\beta}} + \underbrace{\frac{\sin\beta}{\cos\beta}}_{\tan\beta}\frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}}}{\cancel{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}} - \underbrace{\frac{\sin\alpha}{\cos\alpha}}_{\tan\alpha}\underbrace{\frac{\sin\beta}{\cos\beta}}_{\tan\beta}} =$$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \longleftarrow$$

Devono esistere $\tan\alpha$, $\tan\beta$, $\tan(\alpha+\beta)$ e $1-\tan\alpha\tan\beta \neq 0$

$$\alpha \neq \frac{\pi}{2} + k\pi, \quad \beta \neq \frac{\pi}{2} + k\pi, \quad \alpha+\beta \neq \frac{\pi}{2} + k\pi$$

Sotto queste condizioni si ha sicuramente che $1-\tan\alpha\tan\beta \neq 0$. Infatti, se per assurdo fosse $1-\tan\alpha\tan\beta = 0$, si avrebbe $\tan\alpha\tan\beta = 1$, da cui

$$\tan\alpha = \frac{1}{\tan\beta}, \text{ cioè } \boxed{\tan\alpha = \cot\beta}.$$

$\neq 0$ perché $\tan\alpha\tan\beta = 1$

↓ QUANDO VALE?

Sappiamo che $\cot\beta = \tan\left(\frac{\pi}{2} - \beta\right)$, dunque

$$\tan\alpha = \tan\left(\frac{\pi}{2} - \beta\right) \iff \alpha = \frac{\pi}{2} - \beta + k\pi$$

$$\alpha + \beta = \frac{\pi}{2} + k\pi$$

CONTRO L'IPOTESI DI PARTENZA