16/10/2018

$$\sin 15° = \sin(45° - 30°) =$$

$$= \sin 45° \cos 30° - \sin 30° \cos 45° =$$

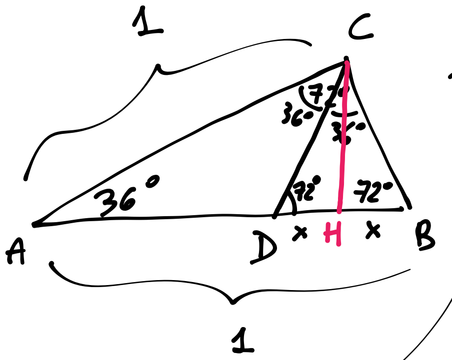
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos 75° = \cos(30° + 45°) =$$

$$= \cos 30° \cos 45° - \sin 30° \sin 45° =$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$36° \rightsquigarrow \frac{\pi}{5}$$



ABC simile CBD

$$\boxed{\overline{CB} : 1 = \overline{DB} : \overline{CB}}$$

$\overline{HB} = x$

$x^2 + \overline{CH}^2 = \overline{CB}^2$

$\overline{AH}^2 + \overline{CH}^2 = 1$

$(\overline{AD} + x)^2 + \overline{CH}^2 = 1$

$(\overline{CB} + x)^2 + \overline{CH}^2 = 1$

$\overline{AD} = \overline{CD} = \overline{CB}$

$\overline{CB}^2 = \overline{DB} = 2x$

$\overline{CH}^2 = \overline{CB}^2 - x^2 = 2x - x^2$

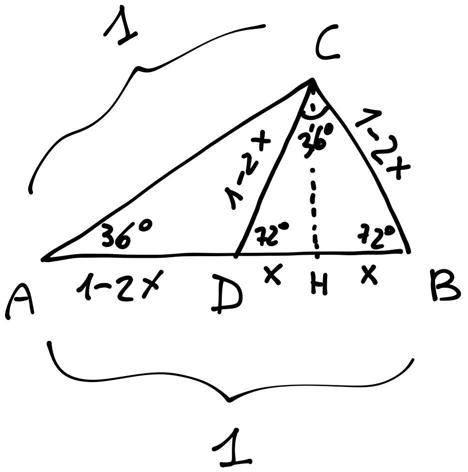
$\overline{CB} = \sqrt{2x}$

$(\sqrt{2x} + x)^2 + 2x - x^2 = 1$

$2x + \cancel{x^2} + 2\sqrt{2}\, x \sqrt{x} + 2x - \cancel{x^2} = 1$

$4x + 2\sqrt{2}\, x \sqrt{x} - 1 = 0$

GIUSTO! MA COSÌ È DIFFICILE DA RISOLVERE



$\overline{AD} = \overline{DC} = \overline{CB}$

$\hat{ABC} \cong \hat{CBD}$ SIMILE

$\Downarrow$

$\overline{AC} : \overline{CB} = \overline{CB} : \overline{DB}$

$1 : (1-2x) = (1-2x) : (2x)$

$\Downarrow$

$(1-2x)^2 = 2x$

$1 + 4x^2 - 4x - 2x = 0$

$4x^2 - 6x + 1 = 0$

$x = \frac{3 \pm \sqrt{9-4}}{4} = \frac{3 \pm \sqrt{5}}{4} =$

$\frac{3+\sqrt{5}}{4} > 1$ NON ACC.

$\frac{3-\sqrt{5}}{4} < 1$ OK

$\overline{AH} = \cos 36° = 1-2x+x = 1-x = 1 - \frac{3-\sqrt{5}}{4} = \frac{4-3+\sqrt{5}}{4} = \boxed{\frac{\sqrt{5}+1}{4}}$

$\sin 36° = \sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)^2} = \sqrt{1-\frac{5+1+2\sqrt{5}}{16}} = \sqrt{\frac{10-2\sqrt{5}}{16}} =$

$= \boxed{\frac{\sqrt{10-2\sqrt{5}}}{4}}$

$$\sin 2\alpha = \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha =$$
$$= 2\sin\alpha\cos\alpha$$
$$\Downarrow$$
$$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$$

$$\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cdot\cos\alpha - \sin\alpha\cdot\sin\alpha =$$
$$= \boxed{\cos^2\alpha - \sin^2\alpha} =$$
$$= 1 - \sin^2\alpha - \sin^2\alpha = \boxed{1 - 2\sin^2\alpha} =$$
$$= 1 - 2(1-\cos^2\alpha) = 1 - 2 + 2\cos^2\alpha =$$
$$= \boxed{2\cos^2\alpha - 1}$$

FORMULE DI DUPLICAZIONE

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$= 2\cos^2\alpha - 1$$
$$= 1 - 2\sin^2\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\alpha \neq \frac{\pi}{4} + k\frac{\pi}{2} \qquad \alpha \neq \frac{\pi}{2} + k\pi$$

124 $\frac{\sin 2\alpha}{1+\cos 2\alpha} - \tan\alpha =$ [0]

$$= \frac{2\sin\alpha\cos\alpha}{\cancel{1} + 2\cos^2\alpha - \cancel{1}} - \tan\alpha =$$

$$= \frac{\cancel{2}\sin\alpha\cancel{\cos\alpha}}{\cancel{2}\cos^{\cancel{2}}\alpha} - \tan\alpha = 0$$

131 $2\cos\alpha\cdot(1+\cos 2\alpha) - \sin\alpha\sin 2\alpha =$

$$= 2\cos\alpha\left(\cancel{1} + 2\cos^2\alpha - \cancel{1}\right) - \sin\alpha\cdot 2\sin\alpha\cos\alpha =$$

$1 - \cos^2\alpha$

$$= 4\cos^3\alpha - 2\sin^2\alpha\cos\alpha = 2\cos\alpha\left(2\cos^2\alpha - \sin^2\alpha\right) =$$

$$= 2\cos\alpha\left(2\cos^2\alpha - 1 + \cos^2\alpha\right) = 2\cos\alpha\left(3\cos^2\alpha - 1\right)$$

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$$\frac{\tan 2\alpha}{\tan\alpha} = \frac{\cos 2\alpha + 1}{\cos 2\alpha}$$

Verificare questa identità (supponendo che le funzioni coinvolte abbiano senso per α)

$$\frac{\dfrac{2\cancel{\tan\alpha}}{1-\tan^2\alpha}}{\cancel{\tan\alpha}} = \frac{2\cos^2\alpha \cancel{-1} + \cancel{1}}{\cos 2\alpha}$$

$$\frac{\cancel{2}}{1-\tan^2\alpha} = \frac{\cancel{2}\cos^2\alpha}{\cos 2\alpha}$$

$$\frac{1}{1-\dfrac{\sin^2\alpha}{\cos^2\alpha}} = \frac{\cos^2\alpha}{\cos 2\alpha}$$

$$\frac{1}{\dfrac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha}} = \frac{\cos^2\alpha}{\cos 2\alpha} \Rightarrow \frac{\cos^2\alpha}{\cos^2\alpha - \sin^2\alpha} = \frac{\cos^2\alpha}{\cos 2\alpha}$$

$$\Downarrow$$

$$\boxed{\frac{\cos^2\alpha}{\cos 2\alpha} = \frac{\cos^2\alpha}{\cos 2\alpha}}$$