24/10/2018

FORMULE DI BISEZIONE

$\cos\frac{\alpha}{2} = ?$ $\sin\frac{\alpha}{2} = ?$

DUPLICAZIONE $\Rightarrow$ $\cos 2\alpha = 2\cos^2\alpha - 1$

$2\alpha = \beta$

$\alpha = \frac{\beta}{2}$

$\Downarrow$

$\cos\beta = 2\cos^2\frac{\beta}{2} - 1$

$\cos\frac{\beta}{2} = \pm\sqrt{\frac{\cos\beta + 1}{2}}$

$$\boxed{\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}}$$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\cos\beta = 1 - 2\sin^2\frac{\beta}{2}$

$\sin\frac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{2}}$

$$\boxed{\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}}$$

$$\boxed{\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}$$

$\downarrow$

$\frac{\alpha}{2} \neq \frac{\pi}{2} + k\pi$

$\alpha \neq \pi + 2k\pi$

221 $\tan\frac{\alpha}{2} + 2\cos^2\frac{\alpha}{2}\cdot\csc\alpha =$ $\left[\frac{2}{\sin\alpha}\right]$

$$= \tan\frac{\alpha}{2} + \cancel{2}\,\frac{1+\cos\alpha}{\cancel{2}}\cdot\frac{1}{\sin\alpha} =$$

$$= \frac{1-\cos\alpha}{\sin\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = \frac{1-\cancel{\cos\alpha}+1+\cancel{\cos\alpha}}{\sin\alpha} = \boxed{\frac{2}{\sin\alpha}}$$

227 $2\sin^2\frac{\alpha}{2}\cdot\cot^2\frac{\alpha}{2} - \cos^2\frac{\alpha}{2} =$ $\left[\frac{1+\cos\alpha}{2}\right]$

$$= \cancel{2}\,\frac{1-\cos\alpha}{\cancel{2}}\cdot\frac{1}{\tan^2\frac{\alpha}{2}} - \frac{1+\cos\alpha}{2} =$$

$$= \frac{1-\cos\alpha}{\frac{1-\cos\alpha}{1+\cos\alpha}} - \frac{1+\cos\alpha}{2} = 1+\cos\alpha - \frac{1+\cos\alpha}{2} =$$

$$= \frac{1+\cos\alpha}{2}$$

205 TEST $\sin\frac{7}{12}\pi$ vale:

A $\frac{\sqrt{3}}{2}$.

B $-\frac{\sqrt{3}}{2}$.

~~C~~ $\frac{\sqrt{2+\sqrt{3}}}{2}$.

D $-\frac{\sqrt{2+\sqrt{3}}}{2}$.

E $\frac{\sqrt{2-\sqrt{3}}}{2}$.

$$\sin\left(\frac{7}{12}\pi\right) = \sin\left(\frac{1}{2}\cdot\underbrace{\frac{7}{6}\pi}_{\alpha}\right) = \sqrt{\frac{1-\cos\left(\frac{7}{6}\pi\right)}{2}} = (*)$$

$$\cos\left(\frac{7}{6}\pi\right) = \cos\left(\pi+\frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$(*) = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

231 $\frac{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos\alpha} - \sin^2\frac{\alpha}{2} = \frac{\tan\alpha}{2} - \tan\frac{\alpha}{2}\cdot\frac{\sin\alpha}{2}$

IDENTITÀ

$$\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos\alpha} - \frac{1-\cos\alpha}{2} = \frac{\sin\alpha}{2\cos\alpha} - \frac{1-\cos\alpha}{\cancel{\sin\alpha}}\cdot\frac{\cancel{\sin\alpha}}{2}$$

$$\frac{\sin\alpha}{2\cos\alpha} - \frac{1-\cos\alpha}{2} = \frac{\sin\alpha}{2\cos\alpha} - \frac{1-\cos\alpha}{2}$$

FORMULE PARAMETRICHE

$$\sin\alpha = \frac{2t}{1+t^2} \qquad \cos\alpha = \frac{1-t^2}{1+t^2}$$

$$t = \tan\frac{\alpha}{2} \qquad \alpha \neq \pi + 2k\pi$$

$$\sin\alpha = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}} = \frac{\dfrac{2\sin\frac{\alpha}{2}\cancel{\cos\frac{\alpha}{2}}}{\cos^{\cancel{2}}\frac{\alpha}{2}}}{\dfrac{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}} =$$

$$= \frac{2\,\dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}{\dfrac{\cancel{\cos^2\frac{\alpha}{2}}}{\cancel{\cos^2\frac{\alpha}{2}}}+\dfrac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}} = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{2t}{1+t^2}$$

$$\cos\alpha = \frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}} = \frac{1-t^2}{1+t^2} \qquad \tan\alpha = \frac{2t}{1-t^2}$$

DIVIDO SOPRA E SOTTO PER $\cos^2\frac{\alpha}{2}$

PICCOLA PUNTUALIZZAZIONE

$$\frac{\pi}{7} \xrightarrow{\text{GRADI}} \qquad x = \frac{\cancel{\pi}}{7} \cdot \frac{180^\circ}{\cancel{\pi}} = 25{,}71428571\ldots^\circ$$

$$\pi : 180^\circ = \frac{\pi}{7} : x$$

$$1^\circ = 60' \qquad 1' = 60''$$

$$1^\circ : 60' = 0{,}71428\ldots^\circ : x$$

$$x = 0{,}71428\ldots \cdot 60' = 42{,}85714\ldots'$$

per i secondi $\quad 0{,}85714\ldots \cdot 60'' = 51{,}42\ldots'' \simeq 51''$

$$\simeq 25^\circ\ 42'\ 51''$$

Scrivere l'espressione in funzione di $t = \tan\frac{\alpha}{2}$

278 $2\tan\alpha + \frac{\cos\alpha}{1+\sin\alpha} =$ $\left[\frac{1+t}{1-t}\right]$

$$= 2\,\frac{2t}{1-t^2} + \frac{\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}} = \frac{4t}{1-t^2} + \frac{\frac{1-t^2}{\cancel{1+t^2}}}{\frac{1+t^2+2t}{\cancel{1+t^2}}} =$$

$$= \frac{4t}{1-t^2} + \frac{1-t^2}{1+t^2+2t} = \frac{4t}{(1-t)(1+t)} + \frac{1-t^2}{(1+t)^2} =$$

$$= \frac{4t(1+t)+(1-t)(1-t^2)}{(1-t)(1+t)^2} = \frac{4t+4t^2+1-t^2-t+t^3}{(1-t)(1+t)^2} =$$

$$= \frac{t^3+3t^2+3t+1}{(1-t)(1+t)^2} = \frac{(t+1)^{\cancel{3}}}{(1-t)\cancel{(1+t)^2}} = \boxed{\frac{1+t}{1-t}}$$

326 **YOU & MATHS** Prove that

$$\sin^2(x+\alpha)+\sin^2(x+\beta)-2\cos(\alpha-\beta)\overbrace{\sin(x+\alpha)\sin(x+\beta)}^{\text{FORMULE DI WERNER}} =$$

is a constant function of x.

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$$= \Big[\underbrace{\sin(x+\alpha)+\sin(x+\beta)}_{\text{FORMULE DI PROSTAFERESI}}\Big]^2 - 2\underbrace{\sin(x+\alpha)\sin(x+\beta)}_{\text{FORMULE DI WERNER}} -$$

$$-\cancel{2}\cos(\alpha-\beta)\cdot\frac{1}{\cancel{2}}\left[\cos(\alpha-\beta)-\cos(2x+\alpha+\beta)\right] =$$

$$= \left[2\sin\frac{2x+\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right]^2 - \cancel{2}\cdot\frac{1}{\cancel{2}}\left[\cos(\alpha-\beta)-\cos(2x+\alpha+\beta)\right]$$

$$-\cos^2(\alpha-\beta)+\cos(\alpha-\beta)\cos(2x+\alpha+\beta) =$$

$$= \overbrace{4\sin^2\frac{2x+\alpha+\beta}{2}\cdot\cos^2\frac{\alpha-\beta}{2}}^{\text{FORMULE DI BISEZIONE}} - \cos(\alpha-\beta)+\cos(2x+\alpha+\beta)$$

$$-\cos^2(\alpha-\beta)+\cos(\alpha-\beta)\cos(2x+\alpha+\beta) =$$

$$= \cancel{4}\cdot\frac{1-\cos(2x+\alpha+\beta)}{\cancel{2}}\cdot\frac{1+\cos(\alpha-\beta)}{\cancel{2}} - \cos(\alpha-\beta)+\cos(2x+\alpha+\beta)$$

$$-\cos^2(\alpha-\beta)+\cos(\alpha-\beta)\cos(2x+\alpha+\beta) =$$

$$= 1+\cancel{\cos(\alpha-\beta)}-\cancel{\cos(2x+\alpha+\beta)}-\cancel{\cos(\alpha-\beta)\cos(2x+\alpha+\beta)}-\cancel{\cos(\alpha-\beta)}$$

$$+\cancel{\cos(2x+\alpha+\beta)}-\cos^2(\alpha-\beta)+\cancel{\cos(\alpha-\beta)\cos(2x+\alpha+\beta)} =$$

$$= 1-\cos^2(\alpha-\beta) = \boxed{\sin^2(\alpha-\beta)} \leftarrow \text{COSTANTE, NON DIPENDE DA } x$$