7/11/2018

EQUAZIONI LINEARI

DEFINIZIONE

Un'**equazione goniometrica lineare in sin x e cos x** si può ricondurre alla forma:

$$a \sin x + b \cos x + c = 0, \quad \text{con } a, b, c \in \mathbb{R}, a \neq 0 \text{ e } b \neq 0.$$

$a = 1 \quad b = -1 \quad c = 0$

249 $\sin x - \cos x = 0$

$\frac{\sin x}{\cos x} - 1 = 0 \qquad \tan x = 1 \Rightarrow \boxed{x = \frac{\pi}{4} + k\pi}$

DIVIDO per $\cos x$

$\cos x \neq 0$. Infatti non può essere $\cos x = 0$, perché altrimenti l'equazione diventerebbe

CONDIZIONE SUPERFLUA, NON NECESSARIA

$\sin x - 0 = 0$

$\Downarrow$

$\sin x = 0$ ASSURDO perché sin e cos non possono essere entrambi 0

252 $\cos x - \sqrt{3}\sin x = 1$ $\left[2k\pi; -\frac{2}{3}\pi + 2k\pi\right]$

1° MODO CON LE FORMULE PARAMETRICHE

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$t = \tan\frac{x}{2}$$

$$x \neq \pi + 2k\pi$$

CONTROLLO SE $\pi + 2k\pi$ È SOLUZIONE:

$$\cos\pi - \sqrt{3}\sin\pi \stackrel{?}{=} 1$$

$$-1 - \sqrt{3}\cdot 0 \stackrel{?}{=} 1$$

$-1 = 1$ NO, NON È SOLUZIONE

PROCEDO ALLA SOSTITUZIONE

$$\frac{1-t^2}{1+t^2} - \sqrt{3}\,\frac{2t}{1+t^2} = 1$$

$$\frac{1-t^2-2\sqrt{3}t}{\cancel{1+t^2}} = \frac{1+t^2}{\cancel{1+t^2}}$$

$$-2t^2 - 2\sqrt{3}t = 0 \qquad t^2 + \sqrt{3}t = 0$$

$$t\left(t+\sqrt{3}\right) = 0$$

$$t = 0 \quad \vee \quad t = -\sqrt{3}$$

$$\tan\frac{x}{2} = 0 \quad \vee \quad \tan\frac{x}{2} = -\sqrt{3}$$

$$\frac{x}{2} = k\pi \quad \vee \quad \frac{x}{2} = -\frac{\pi}{3} + k\pi$$

$$\boxed{x = 2k\pi \quad \vee \quad x = -\frac{2}{3}\pi + 2k\pi}$$

$\boxed{2° \text{ MODO}}$

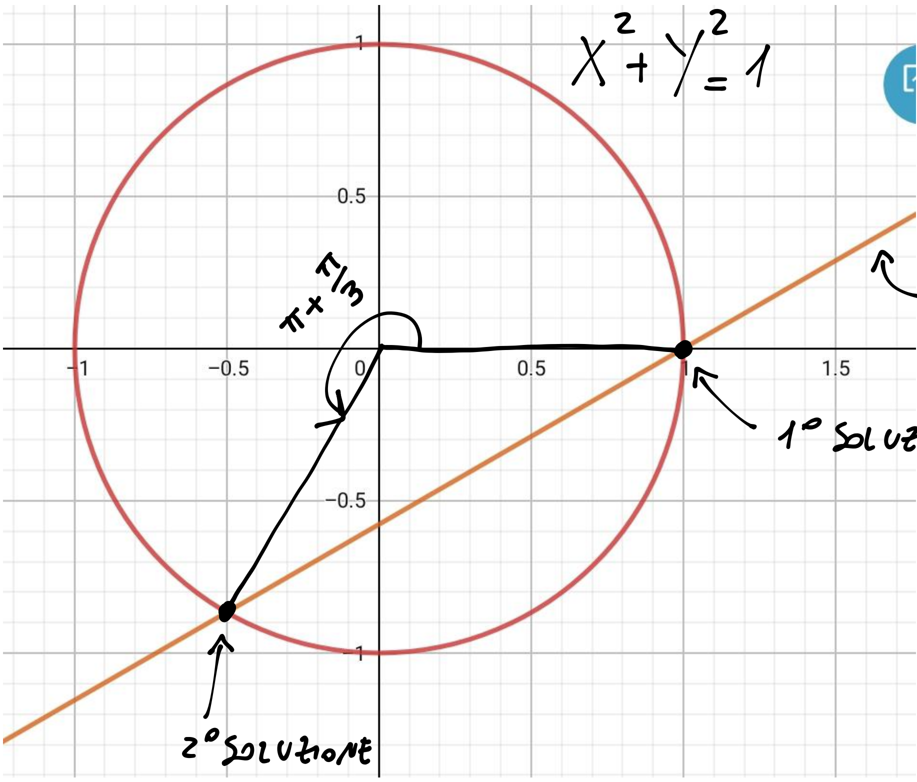
METODO GRAFICO

$$\cos x - \sqrt{3}\sin x = 1$$

$$\begin{cases} X^2 + Y^2 = 1 & \text{EQ. CIRC. GONIOMETRICA} \\ X - \sqrt{3}\,Y = 1 & \text{EQ. DI UNA RETTA} \end{cases}$$

$\cos x = X$

$\sin x = Y$



1° SOLUZIONE $x = 0 + 2k\pi$

$\Downarrow$

$x = 2k\pi$

$$\begin{cases} (1+\sqrt{3}Y)^2 + Y^2 = 1 \\ X = 1 + \sqrt{3}\,Y \end{cases} \quad \begin{cases} \cancel{1} + 3Y^2 + 2\sqrt{3}\,Y + Y^2 = \cancel{1} \\ X = 1 + \sqrt{3}\,Y \end{cases} \quad \begin{cases} 4Y^2 + 2\sqrt{3}\,Y = 0 \\ X = 1 + \sqrt{3}\,Y \end{cases}$$

$$\begin{cases} 2Y^2 + \sqrt{3}\,Y = 0 \\ \dots \end{cases} \quad \begin{cases} Y(2Y + \sqrt{3}) = 0 \\ \end{cases} \Rightarrow \begin{cases} X = 1 \\ Y = 0 \end{cases} \vee \begin{cases} X = -\frac{1}{2} \\ Y = -\frac{\sqrt{3}}{2} \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\boxed{x = 2k\pi \quad \vee \quad x = \frac{4}{3}\pi + 2k\pi}$$

264 $(2+\sqrt{3})\sin x - \cos x + 2 + \sqrt{3} = 0$

$$\begin{cases} X^2 + Y^2 = 1 \\ (2+\sqrt{3})Y - X + 2 + \sqrt{3} = 0 \Rightarrow X = (2+\sqrt{3})Y + 2 + \sqrt{3} \end{cases}$$

$$\left[(2+\sqrt{3})Y + 2 + \sqrt{3}\right]^2 + Y^2 - 1 = 0$$

$$(2+\sqrt{3})^2 Y^2 + (2+\sqrt{3})^2 + 2(2+\sqrt{3})^2 Y + Y^2 - 1 = 0$$

$$(4+3+4\sqrt{3})Y^2 + 4 + 3 + 4\sqrt{3} + (14 + 8\sqrt{3})Y + Y^2 - 1 = 0$$

$$7Y^2 + 4\sqrt{3}Y^2 + 6 + 4\sqrt{3} + 14Y + 8\sqrt{3}Y + Y^2 = 0$$

$$8Y^2 + 4\sqrt{3}Y^2 + 14Y + 8\sqrt{3}Y + 6 + 4\sqrt{3} = 0$$

$$(4+2\sqrt{3})Y^2 + (7+4\sqrt{3})Y + 3 + 2\sqrt{3} = 0$$

$$Y = \frac{-(7+4\sqrt{3}) \pm \sqrt{(7+4\sqrt{3})^2 - 4(4+2\sqrt{3})(3+2\sqrt{3})}}{2(4+2\sqrt{3})} = (*)$$

RADICANDO $= 49 + 48 + 56\sqrt{3} - 4(12 + 8\sqrt{3} + 6\sqrt{3} + 12) =$

$$= 49 + \cancel{48} + \cancel{56\sqrt{3}} - \cancel{48} - \cancel{56\sqrt{3}} - 48 = 1$$

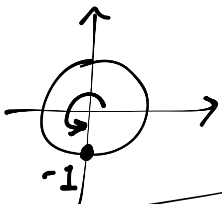
$$(*) = \frac{-7-4\sqrt{3} \pm 1}{2(4+2\sqrt{3})} = \begin{cases} \dfrac{-8-4\sqrt{3}}{2(4+2\sqrt{3})} = \dfrac{\cancel{4}(-2-\sqrt{3})}{\cancel{4}(2+\sqrt{3})} = -1 \\ \dfrac{-6-4\sqrt{3}}{2(4+2\sqrt{3})} = \dfrac{\cancel{2}(-3-2\sqrt{3})}{\cancel{2}(4+2\sqrt{3})} = \dots \end{cases}$$

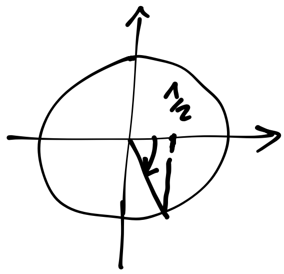
$$Y = \begin{cases} -1 \\ \dfrac{-3-2\sqrt{3}}{4+2\sqrt{3}} \cdot \dfrac{2-\sqrt{3}}{2-\sqrt{3}} = \dfrac{\cancel{-6}+3\sqrt{3}-4\sqrt{3}\cancel{+6}}{2(4-3)} = -\dfrac{\sqrt{3}}{2} \end{cases}$$

$$4+2\sqrt{3} = 2(2+\sqrt{3})$$

$$\begin{cases} X = 0 \\ Y = -1 \end{cases} \qquad \begin{cases} X = (2+\sqrt{3})\left(-\dfrac{\sqrt{3}}{2}\right)+2+\sqrt{3} = \cancel{-\sqrt{3}} - \dfrac{3}{2} + 2 \cancel{+\sqrt{3}} = \dfrac{1}{2} \\ Y = -\dfrac{\sqrt{3}}{2} \end{cases}$$

$$\begin{cases} X = \dfrac{1}{2} \\ Y = -\dfrac{\sqrt{3}}{2} \end{cases}$$





$$x = \frac{3}{2}\pi + 2k\pi \quad \vee \quad x = -\frac{\pi}{3} + 2k\pi$$

262 $\sqrt{3}\,\sin x + \cos x + 1 = 0$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

CONTROLLO $x = \pi + 2k\pi$

$$\sqrt{3}\cdot \sin\pi + \cos\pi + 1 \overset{?}{=} 0$$

$$\sqrt{3}\cdot 0 - 1 + 1 = 0$$ OK, É SOLUZIONE

$$\boxed{x = \pi + 2k\pi}$$

DA AGGIUNGERE ALLA FINE

$$\sqrt{3}\,\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} + 1 = 0$$

$$\frac{2\sqrt{3}\,t + 1 - \cancel{t^2} + 1 + \cancel{t^2}}{\cancel{1+t^2}} = 0$$

$$2\sqrt{3}\,t + 2 = 0$$

$$t = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = -\frac{\sqrt{3}}{3}$$

$$\frac{x}{2} = -\frac{\pi}{6} + k\pi$$

$$\boxed{x = \pi + 2k\pi \quad \vee \quad x = -\frac{\pi}{3} + 2k\pi}$$

288 $3\sin^2 x + 2\sqrt{3}\,\sin x \cos x + \cos^2 x = 0$

È UN QUADRATO

$$\left(\sqrt{3}\sin x + \cos x\right)^2 = 0$$

$$\Rightarrow \quad \sqrt{3}\sin x + \cos x = 0$$

.....

E SE NON MI ACCORGO CHE È UN QUADRATO?

$$3\frac{\sin^2 x}{\cos^2 x} + 2\sqrt{3}\,\frac{\sin x\,\cancel{\cos x}}{\cancel{\cos^2} x} + \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 0$$

POSSO DIVIDERE PER $\cos^2 x$

PERCHÉ $\cos x \neq 0$ CERTAMENTE!!!...

$$3\tan^2 x + 2\sqrt{3}\tan x + 1 = 0$$

$$\tan x = \frac{-\sqrt{3} \pm \sqrt{3-3}}{3} = -\frac{\sqrt{3}}{3}$$

$$\boxed{x = -\frac{\pi}{6} + k\pi}$$

292 $4\sin^2 x + 2\sin x\cos x + 4\cos^2 x = 3$

$$3 \cdot \overbrace{(\cos^2 x + \sin^2 x)}^{1}$$

$$4\sin^2 x + 2\sin x\cos x + 4\cos^2 x = 3\cos^2 x + 3\sin^2 x$$

$$\sin^2 x + 2\sin x\cos x + \cos^2 x = 0$$

divido per $\cos^2 x$

$$\tan^2 x + 2\tan x + 1 = 0$$

$$\tan x = \frac{-2 \pm \sqrt{4-4}}{2} = -1$$

$$\boxed{x = -\frac{\pi}{4} + k\pi}$$