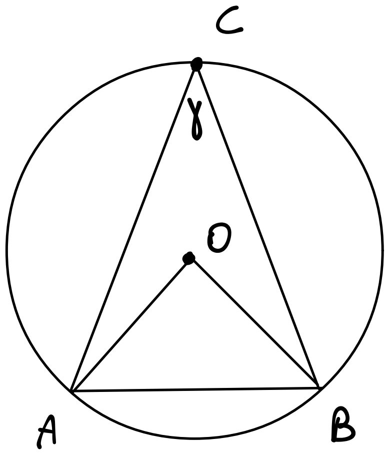
6/12/2018

170 In una circonferenza di raggio 2, la corda AB misura $\frac{16}{9}\sqrt{5}$. Preso C sull'arco maggiore $\widehat{AB}$ in modo che $\overline{AC} = \overline{CB}$, determina il perimetro del triangolo ABC. $\left[\frac{40}{9}\sqrt{5}\right]$



$\overline{AB} = \frac{16}{9}\sqrt{5}$ $\quad r = 2$

$\overline{AB} = 2r \sin\gamma$

$\sin\gamma = \frac{\overline{AB}}{2r} = \frac{\frac{16}{9}\sqrt{5}}{4} = \frac{4}{9}\sqrt{5}$

$\hat{A} = \frac{\pi}{2} - \frac{\gamma}{2}$

$\overline{CB} = 2r\sin\hat{A} = 4\cdot\sin\left(\frac{\pi}{2} - \frac{\gamma}{2}\right) =$

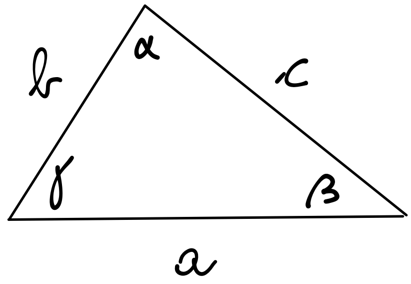
$= 4\cos\frac{\gamma}{2} = 4\sqrt{\frac{1+\cos\gamma}{2}} = \ldots (*)$

$\cos\gamma = \sqrt{1-\sin^2\gamma} = \sqrt{1-\frac{16}{81}\cdot 5} = \sqrt{\frac{81-80}{81}} = \frac{1}{9}$

$(*) \ldots = 4\sqrt{\frac{1+\frac{1}{9}}{2}} = 4\sqrt{\frac{5}{9}} = \frac{4}{3}\sqrt{5}$

$2p = 2\cdot\frac{4}{3}\sqrt{5} + \frac{16}{9}\sqrt{5} = \frac{8}{3}\sqrt{5} + \frac{16}{9}\sqrt{5} = \boxed{\frac{40}{9}\sqrt{5}}$

219 $a = 12, \quad b = 6, \quad \gamma = \frac{\pi}{3}. \qquad c? \qquad [6\sqrt{3}]$



$$c^2 = a^2 + b^2 - 2ab\cos\gamma = 12^2 + 6^2 - 2\cdot 12\cdot 6\cdot\cos\frac{\pi}{3} =$$

$$= 144 + 36 - 72 = 108$$

$$\Downarrow$$

$$c = \sqrt{108} = \sqrt{3^3\cdot 2^2} = 2\cdot 3\cdot\sqrt{3} = \boxed{6\sqrt{3}}$$

RISOLVERE

258 $a = 8\sqrt{6}, \quad \alpha = \frac{2}{3}\pi, \quad \beta = \frac{\pi}{12} \qquad \gamma = ? \quad b = ? \quad c = ?$

$$\gamma = \pi - \frac{2}{3}\pi - \frac{\pi}{12} = \frac{12-8-1}{12}\pi = \frac{\pi}{4}$$

TH. SENI $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \Longrightarrow c = a\cdot\frac{\sin\gamma}{\sin\alpha} = 8\sqrt{6}\cdot\frac{\frac{\sqrt{2}}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}} =$

$$= 8\overset{\sqrt{2}}{\cancel{\sqrt{6}}}\cdot\frac{\sqrt{2}}{\cancel{\sqrt{3}}} = 16$$

TH. COSENO $b^2 = a^2 + c^2 - 2ac\cos\beta = 384 + 256 - \overset{64}{\cancel{256}}\sqrt{6}\cdot\frac{\sqrt{6}+\sqrt{2}}{\cancel{4}} =$

$$= 640 - 384 - 64\sqrt{12} = 256 - 128\sqrt{3}$$

$$b = \sqrt{256 - 128\sqrt{3}} = \sqrt{128}\cdot\sqrt{2-\sqrt{3}} = 2^3\sqrt{2}\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right) = 4\sqrt{2}\left(\sqrt{6}-\sqrt{2}\right) =$$

$$= \boxed{8\sqrt{3} - 8}$$

$$\sqrt{2-\sqrt{3}} = \cancel{\sqrt{}}(a+b)^{\cancel{2}}$$

$$\uparrow$$

OBIETTIVO

$$2-\sqrt{3} = (a+b)^2$$

$$\Downarrow \quad a, b > 0$$

$$\underbrace{2}\,-\underbrace{\sqrt{3}} = (a-\sqrt{3}\,b)^2 = \underbrace{a^2+3b^2}\,\underbrace{-2ab\sqrt{3}}$$

$$\begin{cases} a^2+3b^2 = 2 \\ 2ab = 1 \rightsquigarrow a = \dfrac{1}{2b} \end{cases}$$

$$\frac{1}{4b^2} + 3b^2 = 2$$

$$1 + 12b^4 = 8b^2$$

$$12b^4 - 8b^2 + 1 = 0$$

$$b^2 = \frac{4 \pm \sqrt{16-12}}{12} = \frac{4 \pm 2}{12} = \begin{cases} \frac{1}{2} \\ \frac{1}{6} \end{cases}$$

$$\begin{cases} a = \dfrac{\sqrt{2}}{2} \\ b = \dfrac{\sqrt{2}}{2} \end{cases}$$

$$\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}\right)^2 = \frac{1}{2} + \frac{3}{2} - \cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \frac{\sqrt{6}}{2} = 2 - \sqrt{3}$$

$$\sqrt{2-\sqrt{3}} = \cancel{\sqrt{}}\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)^{\cancel{2}} = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}$$