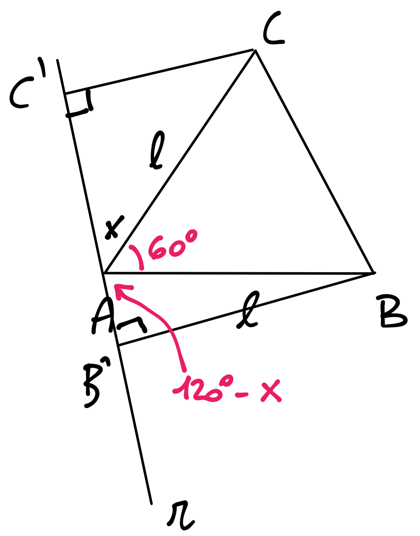
11/12/2018

5. Dato il triangolo equilatero ABC di lato di misura l, conduci dal vertice A una retta r che non attraversi il triangolo in modo che la misura dell'area del trapezio $BCC'B'$ ottenuto proiettando i vertici B e C sulla retta r sia $\frac{l^2}{8}(3+2\sqrt{3})$. $[C\widehat{A}C' = 45° \vee C\widehat{A}C' = 75°]$



$$0° < x < 120°$$

$$\mathcal{A} = \frac{(\overline{B'B} + \overline{C'C})\overbrace{(\overline{C'A} + \overline{AB'})}^{\text{altezza}}}{2}$$

$$\overline{C'C} = l \sin x$$

$$\overline{B'B} = l \sin(120° - x)$$

$$\overline{C'B'} = \overline{C'A} + \overline{AB'} = l\cos x + l\cos(120° - x)$$

$$\mathcal{A} = \frac{(\cancel{l}\sin(120° - x) + \cancel{l}\sin x)(\cancel{l}\cos x + \cancel{l}\cos(120° - x))}{\cancel{2}} = \frac{\cancel{l^2}}{\cancel{8}\,4}(3+2\sqrt{3})$$

$$[\sin 120° \cos x - \cos 120° \sin x + \sin x][\cos x + \cos 120° \cos x + \sin 120° \sin x] =$$

$$= \frac{3+2\sqrt{3}}{4}$$

$$\left[\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x + \sin x\right]\left[\cos x - \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right] = \frac{3+2\sqrt{3}}{4}$$

$$(\sqrt{3}\cos x + 3\sin x)(\cos x + \sqrt{3}\sin x) = 3 + 2\sqrt{3}$$

$$\sqrt{3}\cos^2 x + 3\cos x \sin x + 3\sin x \cos x + 3\sqrt{3}\sin^2 x = (3+2\sqrt{3})\overbrace{\,}^{\cos^2 x + \sin^2 x}$$

$$(\sqrt{3} - 3 - 2\sqrt{3})\cos^2 x + 6\sin x \cos x + (3\sqrt{3} - 3 - 2\sqrt{3})\sin^2 x = 0$$

$$(\sqrt{3} - 3)\sin^2 x + 6\sin x\cos x - (\sqrt{3}+3)\cos^2 x = 0$$

$$(\sqrt{3}-3)\tan^2 x + 6\tan x - (3+\sqrt{3}) = 0$$

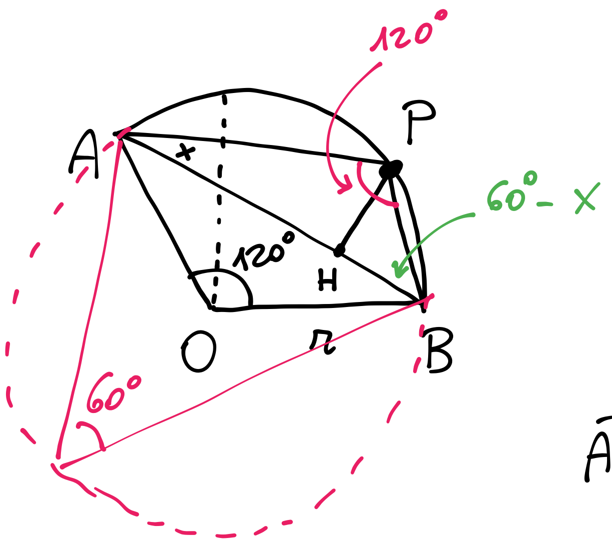
$$\tan x = \frac{-3 \pm \sqrt{9+(3-9)}}{\sqrt{3}-3} = \frac{-3 \pm \sqrt{3}}{\sqrt{3}-3} =$$

$$= \begin{cases} 1 \Rightarrow \boxed{x = 45°} \\ \dfrac{-(\sqrt{3}+3)}{\sqrt{3}-3} \cdot \dfrac{\sqrt{3}+3}{\sqrt{3}+3} = -\dfrac{3+9+6\sqrt{3}}{3-9} = \end{cases}$$

$$= -\frac{12+6\sqrt{3}}{-6} = 2+\sqrt{3} \Rightarrow \boxed{x = 75°}$$

8. Sia AOB un settore circolare di centro O, di raggio $\overline{AO} = \overline{OB} = r$ e di ampiezza $120°$. Determina sull'arco $\widehat{AB}$ un punto P tale che, detta H la proiezione di P sulla corda AB, sia $\overline{AH} + 3\overline{BH} = (2\sqrt{3}+1)r$.

$[P\widehat{A}B = 45°]$



$\overline{AH}$ $\overline{BH}$

DA TROVARE

$0° < x < 60°$

$$\overline{AH} = \overline{AP}\cos x =$$

$$= (2r \cdot \sin(60° - x)) \cdot \cos x$$

$$\overline{HB} = \overline{PB}\cos(60° - x) =$$

$$= (2r \cdot \sin x)\cos(60° - x)$$

$$\overline{AH} + 3\overline{BH} = (2\sqrt{3}+1)r$$

$$2\cancel{r}\cos x(\sin 60° \cos x - \cos 60° \sin x) + 6\cancel{r}\sin x(\cos 60° \cos x +$$

$$+ \sin 60° \sin x) = (2\sqrt{3}+1)\cancel{r}$$

$\cos^2 x + \sin^2 x$

$$\sqrt{3}\cos^2 x - \cos x \sin x + 3\sin x \cos x + 3\sqrt{3}\sin^2 x = (2\sqrt{3}+1)$$

$$(\sqrt{3}+1)\cos^2 x - 2\sin x \cos x + (1-\sqrt{3})\sin^2 x = 0$$

$$(1-\sqrt{3})\tan^2 x - 2\tan x + (1+\sqrt{3}) = 0$$

$$\tan x = \frac{1 \pm \sqrt{1-(-2)}}{1-\sqrt{3}} =$$

$1 \Rightarrow \boxed{x = 45°}$

$\frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$ N.A.