2/10/2018

ESERCIZI Calcolare i seguenti limiti per $n \to \infty$

1) $\dfrac{3n^3 - 5n}{7n^3 + 2n + 1}$ 2) $\dfrac{5}{\sqrt{n^2+2}}$ 3) $7n^2 + 2n - \dfrac{1}{n}$

4) $\sin(n)$ 5) $\sin(n\pi)$

SOLUZIONE

1) $$\lim_{n\to\infty} \frac{3n^3 - 5n}{7n^3 + 2n + 1} = \boxed{\frac{3}{7}}$$

$$\frac{3n^3 - 5n}{7n^3 + 2n + 1} = \frac{n^3\left(3 - \frac{5}{n^2}\right)}{n^3\left(7 + \frac{2}{n^2} + \frac{1}{n^3}\right)} \longrightarrow \frac{3}{7}$$

($\frac{5}{n^2} \to 0$, $\frac{2}{n^2} \to 0$, $\frac{1}{n^3} \to 0$)

2) $$\lim_{n\to\infty} \frac{5}{\sqrt{n^2+2}} = \frac{5}{+\infty} = \boxed{0}$$

($\sqrt{n^2+2} \to +\infty$)

3) $$\lim_{n\to\infty} \left(7n^2 + 2n - \frac{1}{n}\right) = +\infty + \infty - 0 = \boxed{+\infty}$$

($7n^2 \to +\infty$, $2n \to +\infty$, $\frac{1}{n} \to 0$)

$$n^2\left(7 + \frac{2}{n} - \frac{1}{n^3}\right) \longrightarrow +\infty \cdot (7 + 0 - 0) = +\infty \cdot 7 = \boxed{+\infty}$$

($n^2 \to +\infty$, $\frac{2}{n} \to 0$, $\frac{1}{n^3} \to 0$)

4) $\lim_{n\to\infty} \sin(n)$ NON ESISTE

PROVIAMO

$\sin(1000) \cong 0,82687....$

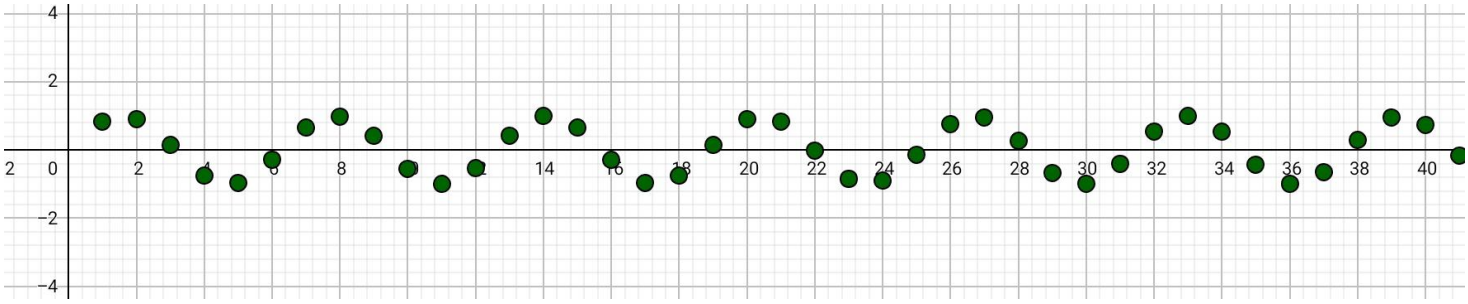
$\sin(1005) \simeq -0,3047....$

$\sin(4518) \simeq 0,37997....$

$\sin(3000000000) \simeq 0,987....$

$\sin(3000000001) \simeq 0,39806....$

$\sin(n)$ è una successione OSCILLANTE



5) $\lim_{n\to\infty} \sin(n\pi) = 0$

$\sin(n\pi) = 0$ per ogni $n \in \mathbb{N}$, quindi $\sin(n\pi)$ è la successione costante 0. Il suo limite per $n \to \infty$ è 0.

FORME INDETERMINATE

1

$$\lim_{n\to\infty} \frac{3n^2}{2n^2+1} = \frac{+\infty}{+\infty}$$

$$\frac{3n^2}{n^2\left(2+\frac{1}{n^2}\right)} \to \frac{3}{2}$$

$\frac{1}{n^2} \to 0$

2

$$\lim_{n\to\infty} \frac{3n}{2n^2+1} = \frac{+\infty}{+\infty}$$

$$\frac{3n}{n^2\left(2+\frac{1}{n^2}\right)} \to \frac{3}{+\infty} = 0$$

$n^2 \to +\infty$, $\frac{1}{n^2} \to 0$, $2+0 \to 2$

3

$$\lim_{n\to\infty} \frac{3n^2}{2n+1} = \frac{+\infty}{+\infty}$$

$$\frac{3n^2}{n\left(2+\frac{1}{n}\right)} \to \frac{+\infty}{2} = +\infty$$

$\frac{1}{n} \to 0$