3/10/2018

FORMA INDETERMINATA $+\infty - \infty$

$[-\infty + \infty]$

1) $\lim\limits_{n\to\infty}\left(n^2 - n\right) = +\infty - \infty$

$$n^2 - n = \underbrace{n^2}_{+\infty}\underbrace{\left(1 - \overset{0}{\frac{1}{n}}\right)}_{1} \to +\infty \cdot 1 = +\infty$$

quindi $\lim\limits_{n\to\infty}\left(n^2 - n\right) = +\infty$

2) ESEMPIO BANALISSIMO

$\lim\limits_{n\to\infty}(n - n) = +\infty - \infty$ ma $\lim\limits_{n\to\infty}(n - n) = \lim\limits_{n\to\infty} 0 = 0$

3) ESEMPIO MENO BANALE

$$\lim_{n\to\infty}\left(\sqrt{n^2+1} - n\right) = +\infty - \infty$$

1° TENTATIVO

$$\sqrt{n^2\left(1+\frac{1}{n^2}\right)} - n = n\sqrt{1+\frac{1}{n^2}} - n = \overset{+\infty}{n}\underbrace{\left[\overset{1}{\sqrt{1+\underset{0}{\frac{1}{n^2}}}} - 1\right]}_{0} \to$$

$\to +\infty \cdot 0$ ALTRA FORMA INDETERMINATA

PICCOLA PARENTESI: perché $\infty \cdot 0$ è forma indeterminata?

$\overset{+\infty}{n} \cdot \overset{0}{\frac{1}{n}} \to 1$ $\qquad$ $\overset{+\infty}{n^2} \cdot \overset{0}{\frac{1}{n}} = n \to +\infty$ $\qquad$ $\overset{+\infty}{n} \cdot \overset{0}{\frac{1}{n^2}} = \frac{1}{n} \to 0$

$$\left(\sqrt{n^2+1} - n\right)\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n} =$$

$$\boxed{(A-B)(A+B)=A^2-B^2}$$

$$= \frac{\cancel{n^2}+1-\cancel{n^2}}{\sqrt{n^2+1}+n} =$$

$$= \frac{1}{\underbrace{\sqrt{n^2+1}}_{+\infty}+\underbrace{n}_{+\infty}} \longrightarrow \frac{1}{+\infty+\infty} = \frac{1}{+\infty} = 0$$

Quindi $\lim\limits_{n\to+\infty}\left(\sqrt{n^2+1}-n\right)=0$

23. $\sqrt{n+1} - \sqrt{n-2}$

$$\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n-2}\right) = +\infty - \infty \quad \text{F. I.}$$

$$\left(\sqrt{n+1} - \sqrt{n-2}\right) \frac{\sqrt{n+1} + \sqrt{n-2}}{\sqrt{n+1} + \sqrt{n-2}} =$$

$$= \frac{n+1-(n-2)}{\sqrt{n+1} + \sqrt{n-2}} = \frac{\cancel{n}+1-\cancel{n}+2}{\sqrt{n+1} + \sqrt{n-2}} = \frac{3}{\underbrace{\sqrt{n+1}}_{+\infty} + \underbrace{\sqrt{n-2}}_{+\infty}} \longrightarrow$$

$$\longrightarrow \frac{3}{+\infty} = 0$$

$$\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n-2}\right) = 0$$

25. $\sqrt{n^2+n}-\sqrt{n^2+3n}$

$$\lim_{n\to\infty}\left(\sqrt{n^2+n}-\sqrt{n^2+3n}\right)=+\infty-\infty \quad \text{F. I.}$$

$$\left(\sqrt{n^2+n}-\sqrt{n^2+3n}\right)\frac{\sqrt{n^2+n}+\sqrt{n^2+3n}}{\sqrt{n^2+n}+\sqrt{n^2+3n}}=$$

$$=\frac{\cancel{n^2}+n-\cancel{n^2}-3n}{\sqrt{n^2+n}+\sqrt{n^2+3n}}=\frac{-2n}{\sqrt{n^2+n}+\sqrt{n^2+3n}}\to\frac{-\infty}{+\infty} \quad \text{F. I.}$$

$$=\frac{-2n}{\sqrt{n^2\left(1+\frac{1}{n}\right)}+\sqrt{n^2\left(1+\frac{3}{n}\right)}}=\frac{-2n}{n\sqrt{1+\frac{1}{n}}+n\sqrt{1+\frac{3}{n}}}=$$

$$=\frac{-2\cancel{n}}{\cancel{n}\left(\sqrt{1+\underbrace{\tfrac{1}{n}}_{\to 0}}+\sqrt{1+\underbrace{\tfrac{3}{n}}_{\to 0}}\right)}\longrightarrow\frac{-2}{\sqrt{1}+\sqrt{1}}=\frac{-2}{2}=\boxed{-1}$$

Quindi $\lim_{n\to\infty}\left(\sqrt{n^2+n}-\sqrt{n^2+3n}\right)=-1$