17/9/2019

371 $y = \sin x + \cos \frac{x}{2}$

Calcolare il periodo

$\sin x \quad T_1 = 2\pi \qquad \cos \frac{x}{2} \quad T_2 = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$T = 4\pi$

372 $y = \tan x + \sin x$

$T_1 = \pi \qquad T_2 = 2\pi$

$T = 2\pi$

373 $y = 2\cos 2x + \sin x$

$T_1 = \frac{2\pi}{2} = \pi \qquad T_2 = 2\pi$

$T = 2\pi$

413 $y = \dfrac{e^{-x} - e^{x}}{e^{2x} - e^{-2x}}$

indicare se pari o dispari ...

$D = \{x \in \mathbb{R} \mid x \neq 0\}$ SIMMETRICO RISP. A 0

$= (-\infty, 0) \cup (0, +\infty)$

$$f(-x) = \frac{e^{x} - e^{-x}}{e^{-2x} - e^{2x}} =$$

$$= \frac{-(-e^{x} + e^{-x})}{-(-e^{-2x} + e^{2x})} = \frac{e^{-x} - e^{x}}{e^{2x} - e^{-2x}} = f(x)$$

PARI

414 $y = \tan^2 x + \sin|x|$

$D = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\}$

$$f(-x) = [\tan(-x)]^2 + \sin|-x| =$$

$$= [-\tan x]^2 + \sin|x| = \tan^2 x + \sin|x| = f(x)$$

PARI

Dimostrare che se una funzione $f: D \to \mathbb{R}$ (con D simmetrico rispetto a 0) è pari e dispari, allora

$$\forall x \in D \qquad f(x) = 0$$

DIMOSTRAZIONE

$$\begin{cases} \boxed{1} \quad f(x) = f(-x) & \text{PARI} \\ \boxed{2} \quad f(-x) = -f(x) & \text{DISPARI} \end{cases} \qquad \forall x \in D$$

$$\forall x \in D \qquad \Rightarrow \underbrace{f(x) = \overbrace{f(-x)}_{}}_{\boxed{1}} \overbrace{= -f(x)}^{\boxed{2}} \Rightarrow f(x) = -f(x)$$

$$\Downarrow$$

$$f(x) + f(x) = 0$$

$$2f(x) = 0$$

$$f(x) = 0 \qquad \text{C.V.D.}$$

411 $y = \dfrac{|x| + x^2}{2x}$

412 $y = \arcsin x + 2x^3$

Controllare parità o disparità

1] $f(x) = \dfrac{|x| + x^2}{2x}$ $\qquad D = (-\infty, 0) \cup (0, +\infty)$

$$f(-x) = \frac{|-x| + (-x)^2}{2(-x)} = \frac{|x| + x^2}{-2x} = -\frac{|x| + x^2}{2x} = -f(x)$$

DISPARI

2] $f(x) = \arcsin x + 2x^3$ $\qquad D = [-1, 1]$

$$f(-x) = \arcsin(-x) + 2(-x)^3 = -\arcsin x - 2x^3 =$$

$$= -\left[\arcsin x + 2x^3\right] = -f(x)$$ DISPARI

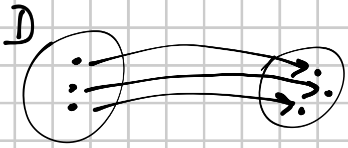
CALCOLARE LA FUNZIONE INVERSA (DOPO AVER VERIFICATO L'INVERTIBILITÀ)

431 $f(x) = 2^{x+3} + 4$ $\qquad [f^{-1}(x) = \log_2(x-4) - 3]$

INVERTIBILE = INIETTIVA $\qquad f: \mathbb{R} \to \mathbb{R}$

$\downarrow$

$\forall x_1, x_2 \in D \qquad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$



$$2^{x_1+3} + 4 = 2^{x_2+3} + 4$$

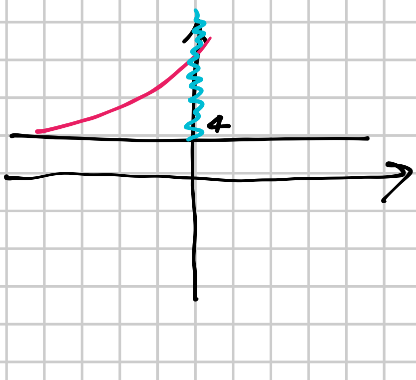
$$2^{x_1+3} = 2^{x_2+3}$$

$$x_1 + 3 = x_2 + 3$$

$x_1 = x_2$ quindi è iniettiva

$f: \mathbb{R} \to \mathbb{R}$

$f(x) = 2^{x+3} + 4$ $\qquad$ INSIEME IMMAGINE $\quad \text{im} f = (4, +\infty)$



$f^{-1}: (4, +\infty) \to \mathbb{R}$

INVERSA

$$y = 2^{x+3} + 4$$

$\Downarrow$ GRAFICO SIMMETRICO RISP. $y = x$

$$x = 2^{y+3} + 4$$

$2^{y+3} = x - 4 \qquad y + 3 = \log_2(x-4) \qquad y = \log_2(x-4) - 3$

$$\boxed{f^{-1}(x) = \log_2(x-4) - 3}$$