23/9/2019

1. $5 + \dfrac{1}{n}$ [5]

2. $\dfrac{3}{n^2}$ [0]

3. $\dfrac{5-n}{2n+1}$ $\left[-\dfrac{1}{2}\right]$

4. $2n^2 + 6n - 1$ $[+\infty]$

5. $-n^3 + 2n^2 - 4$ $[-\infty]$

6. $3 - \dfrac{2}{5n+1}$ [3]

Calcolare i limiti (per $n \to +\infty$) delle successioni

1) $\lim\limits_{n \to +\infty} \left(5 + \underbrace{\frac{1}{n}}_{\downarrow\, 0}\right) = 5$

2) $\lim\limits_{n \to +\infty} \dfrac{3}{n^2} = 0$

3) $\lim\limits_{n \to +\infty} \dfrac{5-n}{2n+1} = \dfrac{-\infty}{+\infty}$

FORMA INDETERMINATA → non posso decidere subito qual è il risultato del limite! Devo trovare un'altra strada...

$$\lim_{n \to +\infty} \frac{5-n}{2n+1} = \lim_{n \to +\infty} \frac{\cancel{n}\left(\overset{\nearrow 0}{\frac{5}{n}} - 1\right)}{\cancel{n}\left(2 + \underbrace{\frac{1}{n}}_{\downarrow\, 0}\right)} = \frac{-1}{2} = -\frac{1}{2}$$

PICCOLA VERIFICA SPERIMENTALE

$$n = 10\,000 \Rightarrow \frac{5 - 10\cdot000}{2 \cdot 10000 + 1} = -\frac{9\cdot 995}{20\,001} = -0{,}49972\ldots \simeq -\frac{1}{2}$$

SULLE FORME INDETERMINATE

Perché $\frac{\infty}{\infty}$ è una F.I.?

Consideriamo le tre successioni

$$1)\ a_n = \frac{n^2}{n} \qquad 2)\ b_n = \frac{n}{n^2} \qquad 3)\ c_n = \frac{5n}{n}$$

Tutte e tre sono forme indeterminate del tipo $\frac{+\infty}{+\infty}$

$$1)\ \lim_{n\to+\infty} \frac{n^2}{n} = \lim_{n\to+\infty} n = +\infty$$

$$2)\ \lim_{n\to+\infty} \frac{n}{n^2} = \lim_{n\to+\infty} \frac{1}{n} = 0 \qquad 3)\ \lim_{n\to+\infty} \frac{5n}{n} = \lim_{n\to+\infty} 5 = 5$$

Le tre successioni hanno limiti diversi: in presenza di forme indeterminate non si può stabilire subito il limite, ma bisogna trovare un'altra strada.

$$4)\ \lim_{n\to+\infty} (2n^2 + 6n - 1) = +\infty + \infty - 1 = +\infty$$

$$5)\ \lim_{n\to+\infty} (-n^3 + 2n^2 - 4) = \underbrace{-\infty + \infty}_{\text{F.I.}} - 4$$

$$= \lim_{n\to+\infty} \underset{\downarrow\ +\infty}{n^3} \left(-1 + \underset{\downarrow\ 0}{\underbrace{\frac{2}{n}}} - \underset{\downarrow\ 0}{\underbrace{\frac{4}{n^3}}}\right) = +\infty \cdot (-1) = -\infty$$

6. $3 - \dfrac{2}{5n+1}$ [3]

7. $\dfrac{(-1)^n}{n+1}$ [0]

8. 2^{-n} [0]

6) $$\lim_{n\to+\infty}\left(3-\underbrace{\frac{2}{5n+1}}_{\downarrow\ 0}\right)=3$$

7) $$\lim_{n\to+\infty}\frac{(-1)^n}{n+1}$$

Applico il TH. DEI 2 CARABINIERI $\quad a_n = \dfrac{(-1)^n}{n+1}$

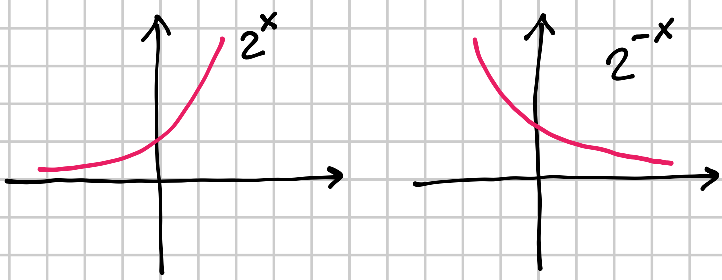
$$c'_n = \frac{-1}{n+1} \qquad\qquad c''_n = \frac{1}{n+1}$$

$$\forall n \in \mathbb{N} \qquad \underset{\displaystyle\downarrow\atop\displaystyle 0}{\frac{-1}{n+1}} \leq \frac{(-1)^n}{n+1} \leq \underset{\displaystyle\downarrow\atop\displaystyle 0}{\frac{1}{n+1}}$$

Dunque existe il limite di $\dfrac{(-1)^n}{n+1}$ e vale 0] Per il TH. DEI 2 CARABINIERI

$$\lim_{n\to+\infty}\frac{(-1)^n}{n+1}=0$$

8) $$\lim_{n\to+\infty} 2^{-n} = 0$$ Bisogna ricordare i grafici elementari!



10. $\dfrac{n^2-5n+1}{4n^2-3n+5}$ $\left[\dfrac{1}{4}\right]$

11. $\dfrac{3n-7}{8n^2+4n+5}$ $[0]$

10) $$\lim_{n\to+\infty} \frac{n^2-5n+1}{4n^2-3n+5} = \frac{+\infty-\infty+1}{+\infty-\infty+5} \quad \text{F.I.}$$

$$= \lim_{n\to+\infty} \frac{\cancel{n^2}\left(1-\overbrace{\frac{5}{n}}^{\to 0}+\overbrace{\frac{1}{n^2}}^{\to 0}\right)}{\cancel{n^2}\left(4-\underbrace{\frac{3}{n}}_{\to 0}+\underbrace{\frac{5}{n^2}}_{\to 0}\right)} = \frac{1}{4}$$

11) $$\lim_{n\to+\infty} \frac{3n-7}{8n^2+4n+5} = \frac{+\infty-7}{+\infty+\infty+5} = \frac{+\infty}{+\infty} \quad \text{F.I.}$$

$$= \lim_{n\to+\infty} \frac{\cancel{n}\left(3-\overbrace{\frac{7}{n}}^{\to 0}\right)}{\underbrace{n^{\cancel{2}}}_{\to+\infty}\left(8+\underbrace{\frac{4}{n}}_{\to 0}+\underbrace{\frac{5}{n^2}}_{\to 0}\right)} = \frac{3}{+\infty} = 0$$