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25. $\sqrt{n^2+n}-\sqrt{n^2+3n}$

$$\lim_{n\to+\infty}\left(\sqrt{n^2+n}-\sqrt{n^2+3n}\right)=+\infty-\infty \quad \text{F.I.}$$

$$=\lim_{n\to+\infty}\left(\sqrt{n^2+n}-\sqrt{n^2+3n}\right)\cdot\frac{\sqrt{n^2+n}+\sqrt{n^2+3n}}{\sqrt{n^2+n}+\sqrt{n^2+3n}}=$$

$$=\lim_{n\to+\infty}\frac{\cancel{n^2}+n-\cancel{n^2}-3n}{\sqrt{n^2+n}+\sqrt{n^2+3n}}=\lim_{n\to+\infty}\frac{-2n}{\sqrt{n^2+n}+\sqrt{n^2+3n}}=\frac{-\infty}{+\infty} \quad \text{F.I.}$$

$$=\lim_{n\to+\infty}\frac{-2n}{\sqrt{n^2\left(1+\frac{1}{n}\right)}+\sqrt{n^2\left(1+\frac{3}{n}\right)}}=\lim_{n\to+\infty}\frac{-2n}{n\sqrt{1+\frac{1}{n}}+n\sqrt{1+\frac{3}{n}}}=$$

$$=\lim_{n\to+\infty}\frac{-2\cancel{n}}{\cancel{n}\left[\sqrt{1+\underbrace{\tfrac{1}{n}}_{\to 0}}+\sqrt{1+\underbrace{\tfrac{3}{n}}_{\to 0}}\right]}=\frac{-2}{\sqrt{1}+\sqrt{1}}=-\frac{2}{2}=\boxed{-1}$$

20. $(1-3\sqrt{n})\dfrac{n-1}{n+2}$

$$\lim_{n\to+\infty} \underbrace{(1-3\sqrt{n})}_{-\infty}\underbrace{\frac{n-1}{n+2}}_{1} = -\infty \cdot 1 = -\infty$$

32. $e^{\frac{n^2+1}{n^2-1}}$ [e]

33. $\sqrt{\dfrac{n^2+4}{2n}}$ [$+\infty$]

32) $$\lim_{n\to+\infty} e^{\overbrace{\frac{n^2+1}{n^2-1}}^{\nearrow 1}} = e^1 = e$$

33) $$\lim_{n\to+\infty} \sqrt{\frac{n^2+4}{2n}} = \sqrt{+\infty} = +\infty$$

perché il radicando tende a $+\infty$ poiché il numeratore ha grado superiore