10/10/2019

TEOREMA

Se $\lim_{x \to \alpha} f(x) = l > 0$ e $\lim_{x \to \alpha} g(x) = m$, allora:

$$\lim_{x \to \alpha} [f(x)]^{g(x)} = l^m.$$

37 $\lim_{x \to 0^+} (x+3)^{\frac{1}{x}}$ $[+\infty]$

$$[f(x)]^{g(x)} = e^{\ln[f(x)]^{g(x)}} = e^{g(x) \cdot \ln f(x)}$$

Applicando il teorema si ha

$$\lim_{x \to 0^+} (x+3)^{\frac{1}{x}} = 3^{+\infty} = +\infty$$

243 $\lim_{x \to -2} \frac{2x^3 + 5x^2 - x - 6}{2x^2 + 3x - 2} = \frac{0}{0}$ $\left[-\frac{3}{5}\right]$

	2	5	-1	-6
-2		-4	-2	6
	2	1	-3	//

$(2x^2 + x - 3)(x+2)$

$$2x^2 + 3x - 2 =$$
$$= 2x^2 + 4x - x - 2 =$$
$$= 2x(x+2) - (x+2) =$$
$$= (x+2)(2x-1)$$

$$\lim_{x \to -2} \frac{(2x^2 + x - 3)\cancel{(x+2)}}{(2x-1)\cancel{(x+2)}} = \frac{3}{-5} = -\frac{3}{5}$$

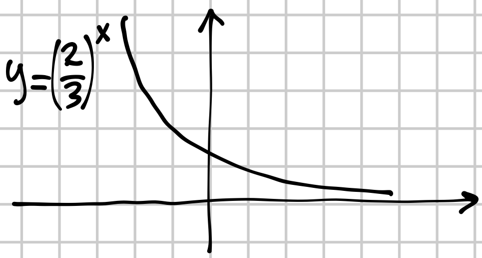
244 $\lim_{x \to 1^+} \left(\frac{3x^2 - 4x + 1}{x^2 + x - 2} \right)^{\frac{1}{1-x}} = \left(\frac{0}{0}\right)^{\infty}$ [$+\infty$]

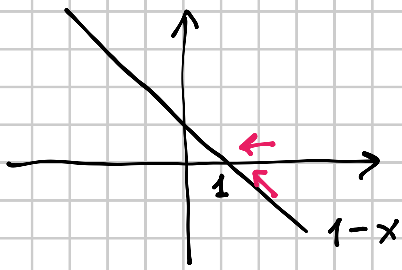
$$\frac{3x^2-4x+1}{x^2+x-2} = \frac{3x^2-3x-x+1}{(x+2)(x-1)} = \frac{3x(x-1)-(x-1)}{(x+2)(x-1)} =$$

$$= \frac{(x-1)(3x-1)}{(x+2)(x-1)}$$

$$\lim_{x \to 1^+} \left(\frac{3x^2-4x+1}{x^2+x-2} \right)^{\frac{1}{1-x}} = \lim_{x \to 1^+} \left(\frac{\cancel{(x-1)}(3x-1)}{(x+2)\cancel{(x-1)}} \right)^{\frac{1}{1-x}} =$$

$$= \lim_{x \to 1^+} \left(\frac{3x-1}{x+2} \right)^{\frac{1}{1-x}} = \left(\frac{2}{3}\right)^{\frac{1}{0^-}} = \left(\frac{2}{3}\right)^{-\infty} = +\infty$$





156 $\lim_{x\to+\infty} \frac{\sqrt{x^2-1}-x}{3x+1} =$ [0]

$$= \lim_{x\to+\infty} \frac{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}-x}{x\left(3+\frac{1}{x}\right)} = \lim_{x\to+\infty} \frac{|x|\sqrt{1-\frac{1}{x^2}}-x}{x\left(3+\frac{1}{x}\right)} =$$

perché $x\to+\infty$

$$= \lim_{x\to+\infty} \frac{x\sqrt{1-\frac{1}{x^2}}-x}{x\left(3+\frac{1}{x}\right)} = \lim_{x\to+\infty} \frac{\cancel{x}\left(\sqrt{1-\frac{1}{x^2}}-1\right)}{\cancel{x}\left(3+\frac{1}{x}\right)} =$$

$\frac{1}{x^2} \to 0$, $\frac{1}{x} \to 0$

$$= \frac{0^-}{3} = 0^-$$

perché $\sqrt{1-\frac{1}{x^2}} < 1$

in un intorno di $+\infty$,

quindi $\sqrt{1-\frac{1}{x^2}}-1 < 0$ in un intorno di $+\infty$

313 $\lim_{x\to 1} \frac{\sqrt{x+3}-\sqrt{5-x}}{\sqrt{1+x}-\sqrt{2}} =$ $[\sqrt{2}]$

$$\frac{\sqrt{x+3}-\sqrt{5-x}}{\sqrt{1+x}-\sqrt{2}} \cdot \frac{\sqrt{x+3}+\sqrt{5-x}}{\sqrt{x+3}+\sqrt{5-x}} =$$

RAZIONALIZZO IL NUMERATORE

RAZIONALIZZO IL DENOMINATORE

$$= \frac{x+3-(5-x)}{\left(\sqrt{x+3}+\sqrt{5-x}\right)\left(\sqrt{1+x}-\sqrt{2}\right)} = \frac{2x-2}{\left(\sqrt{x+3}+\sqrt{5-x}\right)\left(\sqrt{1+x}-\sqrt{2}\right)} \cdot \frac{\sqrt{1+x}+\sqrt{2}}{\sqrt{1+x}+\sqrt{2}}$$

$$= \frac{(2x-2)\left(\sqrt{1+x}+\sqrt{2}\right)}{\left(\sqrt{x+3}+\sqrt{5-x}\right)(1+x-2)} = \frac{2\cancel{(x-1)}\left(\sqrt{1+x}+\sqrt{2}\right)}{\left(\sqrt{x+3}+\sqrt{5-x}\right)\cancel{(x-1)}}$$

$$\xrightarrow[x\to 1]{} \frac{4\sqrt{2}}{2+2} = \sqrt{2}$$