22/10/2019

267 $\lim_{x\to+\infty}(x+1)^{\frac{1}{\ln x}} = \infty^0$ F.I. [e]

$$\lim_{x\to+\infty}(x+1)^{\frac{1}{\ln x}} = \lim_{x\to+\infty} e^{\ln(x+1)^{\frac{1}{\ln x}}} = \lim_{x\to+\infty} e^{\frac{1}{\ln x}\cdot\ln(x+1)} = e$$

$\frac{1}{\ln x}\cdot\ln(x+1) \to 1$ per $x\to+\infty$

A PARTE
$$\frac{\ln(x+1)}{\ln x} = \frac{\ln\left(x\left(1+\frac{1}{x}\right)\right)}{\ln x} = \frac{\ln x + \ln\left(1+\frac{1}{x}\right)}{\ln x} =$$

$$= 1 + \frac{\overbrace{\ln\left(1+\frac{1}{x}\right)}^{0}}{\underbrace{\ln x}_{+\infty}} \longrightarrow 1+0 = 1 \quad \text{per } x\to+\infty$$

269 $\lim_{x\to+\infty} x^{\frac{1}{1+\ln x}} = \infty^0$ F.I. [e]

$$\lim_{x\to+\infty} e^{\ln x^{\frac{1}{1+\ln x}}} = \lim_{x\to+\infty} e^{\frac{1}{1+\ln x}\cdot\ln x} = \lim_{x\to+\infty} e^{\frac{\ln x}{1+\ln x}} = e$$

$\frac{\ln x}{1+\ln x} \to 1$

A PARTE
$$\frac{\ln x}{1+\ln x} = \frac{\cancel{\ln x}}{\cancel{\ln x}\left(\frac{1}{\ln x}+1\right)} = \frac{1}{\underbrace{\frac{1}{\ln x}}_{0}+1} \longrightarrow 1 \quad \text{per } x\to+\infty$$

368 $\lim_{x \to 0} \frac{\cos^2 x - 1}{2x}$ $= \frac{0}{0}$ F.I.

$$\lim_{x \to 0} \frac{\cos^2 x - 1}{2x} = \lim_{x \to 0} \frac{\overbrace{(\cos x - 1)}^{0}\overbrace{(\cos x + 1)}^{2}}{2x} = 0$$

369 $\lim_{x \to 0} \frac{2\tan x + x}{x}$ $= \frac{0}{0}$

$$\lim_{x \to 0} \left[2\frac{\tan x}{x} + \frac{x}{x}\right] = \lim_{x \to 0} \left[2 \overbrace{\frac{\sin x}{x \underbrace{\cos x}_{1}}}^{1} + 1\right] = 2 + 1 = 3$$

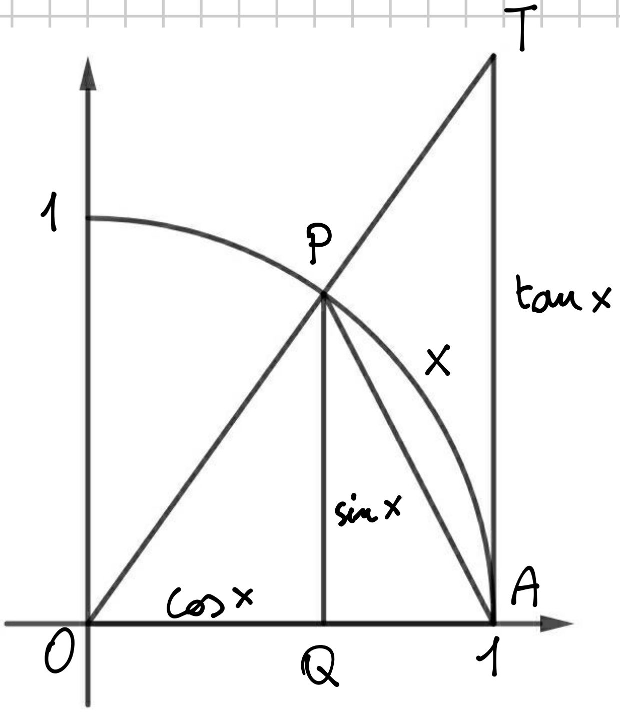
370 $\lim_{x \to 0} \frac{x^2 + x}{2x + \sin x}$ $= \frac{0}{0}$ F.I. $=$

$$= \lim_{x \to 0} \frac{\cancel{x}(x+1)}{\cancel{x}\left(2 + \underbrace{\frac{\sin x}{x}}_{1}\right)} = \frac{1}{3}$$

371 $\lim_{x \to 0} \frac{2x^2}{1 - \cos x}$ $= \lim_{x \to 0} \overbrace{\frac{2x^2}{1 - \cos x}}^{2} = 2 \cdot 2 =$

$= 4$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$



AREA DEL SETTORE CIRCOLARE
(x IN RADIANTI)

$$\underset{\substack{\uparrow\\ \text{AREA}\\ \text{CERCHIO}}}{\pi r^2} : \underset{\substack{\uparrow\\ \text{ANGOLO}\\ \text{AL CENTRO}}}{2\pi} = \underset{\substack{\uparrow\\ \text{AREA}\\ \text{SETTORE}}}{\mathcal{A}_{\text{SETT.}}} : \underset{\substack{\uparrow\\ \text{ANGOLO}\\ \text{AL CENTRO}\\ \text{DEL}\\ \text{SETTORE}}}{x}$$

$$\mathcal{A}_{\text{SETT.}} = \frac{x\cancel{\pi} r^2}{2\cancel{\pi}} = \frac{1}{2}\underbrace{(r x)}_{\text{"BASE"}}\underbrace{r}_{\text{"ALTEZZA"}}$$

$\Downarrow$ $\boxed{r=1}$ CIRC. GON.

$$\mathcal{A}_{\overset{\triangle}{OAP}} < \underset{\substack{\uparrow\\ \text{SETTORE}}}{\mathcal{A}_{\overset{\frown}{OAP}}} < \mathcal{A}_{\overset{\triangle}{OAT}} \implies \frac{1}{2}\sin x < \frac{1}{2}x < \frac{1}{2}\tan x$$

$$0 < x < \frac{\pi}{2} \qquad \Downarrow$$

$$\sin x < x < \tan x$$

Se vale $\sin x < x < \tan x$, allora (dividendo per $\sin x$) si ha

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

passando ai reciproci

$$\cos x < \frac{\sin x}{x} < 1$$

Applico il teorema dei 2 carabinieri per $x \to 0^+$

$$\begin{array}{ccccc} \cos x & < & \frac{\sin x}{x} & < & 1 \\ \downarrow & & \downarrow & & \downarrow \\ 1 & & 1 & & 1 \end{array}$$

Esiste $\lim_{x\to 0^+} \frac{\sin x}{x} = 1$ (si dimostra analogamente per $x \to 0^-$)

Abbiamo dimostrato
$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

DIMOSTRAZIONE

$$\frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{x\,(1+\cos x)} = \frac{\sin^2 x}{x\,(1+\cos x)} = \underbrace{\overbrace{\frac{\sin x}{x}}^{\to 1} \cdot \frac{\overbrace{\sin x}^{\to 0}}{\underbrace{(1+\cos x)}_{\to 2}}}_{\to 0}$$

per $x \to 0$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$ si dimostra esattamente allo stesso modo

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$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

Ricordare che

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$$

DIMOSTRAZIONE

$$\frac{\ln(1+x)}{x} = \ln(1+x)^{\frac{1}{x}}$$

$$\lim_{x\to 0} \ln(1+x)^{\frac{1}{x}} = \ln\left[\lim_{x\to 0}(1+x)^{\frac{1}{x}}\right] = \ln\left[\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^y\right] = (*)$$

per quanto visto nella composizione di funzioni

pongo $\frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

se $x \to 0$, allora $y \to \infty$

$$(*) = \ln e = 1$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

DIMOSTRAZIONE

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim_{y\to 0} \frac{y}{\ln(y+1)} = 1 \quad \text{(reciproco del limite prec.)}$$

pongo $e^x - 1 = y \Rightarrow e^x = y + 1 \Rightarrow x = \ln(y+1)$

se $x \to 0$, allora $y \to 0$

$$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha \qquad \alpha \in \mathbb{R}$$

DIMOSTRAZIONE

$$\frac{(1+x)^\alpha - 1}{x} = \frac{e^{\alpha \ln(1+x)} - 1}{x} \cdot \frac{\alpha \ln(1+x)}{\alpha \ln(1+x)} =$$

$$= \frac{e^{\alpha \ln(1+x)} - 1}{\alpha \ln(1+x)} \cdot \frac{\alpha \ln(1+x)}{x}$$

$$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \lim_{x\to 0} \left[\frac{e^{\alpha \ln(1+x)} - 1}{\alpha \ln(1+x)}\right] \cdot \lim_{x\to 0} \frac{\alpha \ln(1+x)}{x} =$$

$$= \lim_{y\to 0} \frac{e^y - 1}{y} \cdot \lim_{x\to 0} \frac{\alpha \ln(1+x)}{x} = 1 \cdot \alpha \cdot 1 = \alpha$$

$y = \alpha \ln(1+x) \to 0$ per $x \to 0$

430 $\lim_{x\to 0} \frac{\sqrt[6]{1+3x}-1}{x} = \frac{0}{0}$ F.I.

$$\lim_{x\to 0} \frac{(1+x)^{\alpha}-1}{x} = \alpha$$

$$= \lim_{x\to 0} \frac{(1+3x)^{\frac{1}{6}}-1}{x} =$$

$$= \lim_{x\to 0} \underbrace{\frac{(1+3x)^{\frac{1}{6}}-1}{3x}}_{\frac{1}{6}} \cdot 3 = \frac{1}{6} \cdot 3 = \frac{1}{2}$$

449 $\lim_{x\to 0} \frac{(1+2x)^5-1}{5x} \cdot \frac{2}{2} = \lim_{x\to 0} \overbrace{\frac{(1+2x)^5-1}{2x}}^{5} \cdot \frac{2}{5} = 5 \cdot \frac{2}{5} = 2$

441 $\lim_{x\to 4} \frac{\ln(x-3)}{x-4} = \frac{0}{0}$ F.I.

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

$y = x-4$ $\quad y \to 0$ per $x \to 4$ $\quad x-3 = (y+4)-3 = y+1$

$$\lim_{x\to 4} \frac{\ln(x-3)}{x-4} = \lim_{y\to 0} \frac{\ln(y+1)}{y} = 1$$

442 ●●

$$\lim_{x\to 0} \frac{e^{3x}-e^{2x}}{x^2-3x} =$$

$$= \lim_{x\to 0} \frac{\overset{\to 1}{e^{2x}}\,(e^{x}-1)}{x\,\underbrace{(x-3)}_{\to -3}} \to 1 = -\frac{1}{3}$$