

22/10/2019

267 $\lim_{x \rightarrow +\infty} (x+1)^{\frac{1}{\ln x}} = \infty^0$ [e]
F.I.

$$\lim_{x \rightarrow +\infty} (x+1)^{\frac{1}{\ln x}} = \lim_{x \rightarrow +\infty} e^{\ln(x+1)^{\frac{1}{\ln x}}} = \lim_{x \rightarrow +\infty} e^{\frac{1}{\ln x} \cdot \ln(x+1)} = e$$

↑ 1 per $x \rightarrow +\infty$

A PARTE

$$\frac{\ln(x+1)}{\ln x} = \frac{\ln\left(x\left(1+\frac{1}{x}\right)\right)}{\ln x} = \frac{\ln x + \ln\left(1+\frac{1}{x}\right)}{\ln x}$$

$$= 1 + \frac{\overset{0}{\ln\left(1+\frac{1}{x}\right)}}{\underset{+\infty}{\ln x}} \rightarrow 1+0=1 \text{ per } x \rightarrow +\infty$$

269 $\lim_{x \rightarrow +\infty} x^{\frac{1}{1+\ln x}} = \infty^0$ [e]
F.I.

$$\lim_{x \rightarrow +\infty} e^{\ln x^{\frac{1}{1+\ln x}}} = \lim_{x \rightarrow +\infty} e^{\frac{1}{1+\ln x} \cdot \ln x} = \lim_{x \rightarrow +\infty} e^{\frac{\ln x}{1+\ln x}} = e$$

↑ 1

A PARTE

$$\frac{\ln x}{1+\ln x} = \frac{\cancel{\ln x}}{\cancel{\ln x} \left(\frac{1}{\cancel{\ln x}} + 1 \right)} = \frac{1}{\left(\frac{1}{\ln x} + 1 \right)} \rightarrow 1$$

↓ 0 per $x \rightarrow +\infty$

368

$$\lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{2x} = \frac{0}{0} \text{ F.!.}$$

$$\lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{2x} = \lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x + 1)}{2x} = 0$$

369

$$\lim_{x \rightarrow 0} \frac{2 \tan x + x}{x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \left[2 \frac{\tan x}{x} + \frac{x}{x} \right] = \lim_{x \rightarrow 0} \left[2 \frac{\sin x}{x \cos x} + 1 \right] = 2 + 1 = 3$$

370

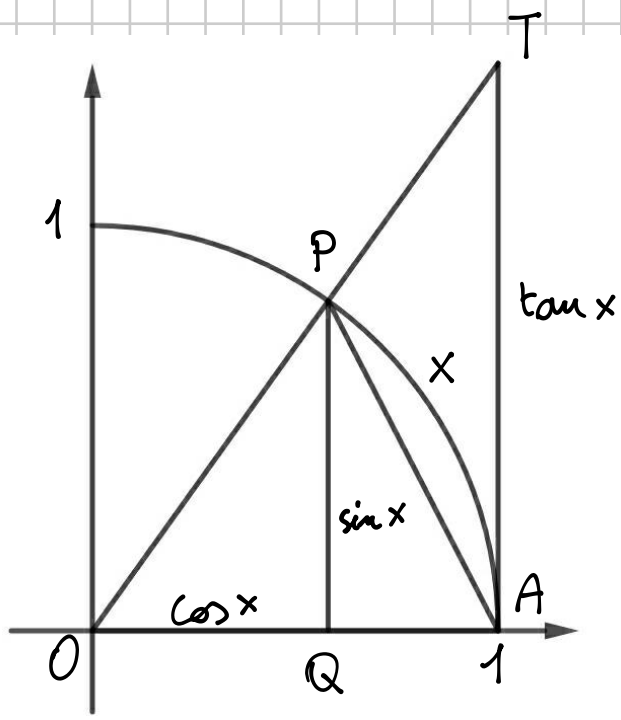
$$\lim_{x \rightarrow 0} \frac{x^2 + x}{2x + \sin x} = \frac{0}{0} \text{ F.!.}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{x} (x + 1)}{\cancel{x} \left(2 + \frac{\sin x}{x} \right)} = \frac{1}{3}$$

371

$$\lim_{x \rightarrow 0} \frac{2x^2}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{2x^2}{1 - \cos x} = 2 \cdot 2 = 4$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$



AREA DEL SETTORE CIRCOLARE
(x IN RADIANTI)

$$\pi r^2 : 2\pi = \mathcal{A}_{\text{SETT.}} : x$$

AREA CIRCLO ANGOLO AL CENTRO AREA SETTORE ANGOLO AL CENTRO DEL SETTORE

$$\mathcal{A}_{\text{SETT.}} = \frac{x \pi r^2}{2\pi} = \frac{1}{2} (\underbrace{rx}_{\text{"BASE"}}) \underbrace{r}_{\text{"ALTEZZA"}}$$

$r=1$ circ. gon.

$$\mathcal{A}_{\triangle OAP} < \mathcal{A}_{\text{SETTORE}} < \mathcal{A}_{\triangle OAT} \Rightarrow \frac{1}{2} \sin x < \frac{1}{2} x < \frac{1}{2} \tan x$$

$$0 < x < \frac{\pi}{2} \quad \Downarrow$$

$$\sin x < x < \tan x$$

Se vale $\sin x < x < \tan x$, allora (dividendo per $\sin x$)
si ha

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

passando ai reciproci

$$\cos x < \frac{\sin x}{x} < 1$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$1 \qquad \qquad \qquad 1$$

Applico il teorema
dei 2 carabinieri per $x \rightarrow 0^+$

Abbiamo dimostrato
 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Esiste $\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$ (si dimostra analogamente per $x \rightarrow 0^-$)

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

DIMOSTRAZIONE

$$\frac{1 - \cos x}{x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{1 - \cos^2 x}{x(1 + \cos x)} = \frac{\sin^2 x}{x(1 + \cos x)} = \frac{\sin x \cdot \sin x}{x(1 + \cos x)}$$

$\begin{matrix} \uparrow 1 & \uparrow 0 \\ \sin x & \cdot \sin x \\ \downarrow 2 \\ x(1 + \cos x) \end{matrix}$
 per $x \rightarrow 0$ \downarrow 0

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \text{ si dimostra esattamente allo stesso modo}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

Ricordare che

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

DIMOSTRAZIONE

$$\frac{\ln(1+x)}{x} = \ln(1+x)^{\frac{1}{x}}$$

$$\lim_{x \rightarrow 0} \ln(1+x)^{\frac{1}{x}} = \ln \left[\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} \right] = \ln \left[\lim_{y \rightarrow \infty} \left(1 + \frac{1}{y}\right)^y \right] = (*)$$

per quanto visto
nella composizione
di funzioni

$$\text{pongo } \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$$

se $x \rightarrow 0$, allora $y \rightarrow \infty$

$$(*) = \ln e = 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

DIMOSTRAZIONE

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{y \rightarrow 0} \frac{y}{\ln(y+1)} = 1 \quad (\text{reciproco del limite prec.})$$

$$\text{per } e^x - 1 = y \Rightarrow e^x = y + 1 \Rightarrow x = \ln(y + 1)$$

se $x \rightarrow 0$, allora $y \rightarrow 0$

$$\lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} = \alpha \quad \alpha \in \mathbb{R}$$

DIMOSTRAZIONE

$$\frac{(1+x)^\alpha - 1}{x} = \frac{e^{\alpha \ln(1+x)} - 1}{x} \cdot \frac{\alpha \ln(1+x)}{\alpha \ln(1+x)} =$$

$$= \frac{e^{\alpha \ln(1+x)} - 1}{\alpha \ln(1+x)} \cdot \frac{\alpha \ln(1+x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} = \lim_{x \rightarrow 0} \left[\frac{e^{\alpha \ln(1+x)} - 1}{\alpha \ln(1+x)} \right] \cdot \lim_{x \rightarrow 0} \frac{\alpha \ln(1+x)}{x} =$$

$$= \lim_{y \rightarrow 0} \frac{e^y - 1}{y} \cdot \lim_{x \rightarrow 0} \frac{\alpha \ln(1+x)}{x} = 1 \cdot \alpha \cdot 1 = \alpha$$

$$y = \alpha \ln(1+x) \rightarrow 0 \text{ per } x \rightarrow 0$$

430

$$\lim_{x \rightarrow 0} \frac{\sqrt[6]{1+3x} - 1}{x} = \frac{0}{0} \quad \text{F.!.}$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} = \alpha$$

$$= \lim_{x \rightarrow 0} \frac{(1+3x)^{\frac{1}{6}} - 1}{x} =$$

$$= \lim_{x \rightarrow 0} \frac{(1+3x)^{\frac{1}{6}} - 1}{3x} \cdot 3 = \frac{1}{6} \cdot 3 = \frac{1}{2}$$

449

$$\lim_{x \rightarrow 0} \frac{(1+2x)^5 - 1}{5x} \cdot \frac{2}{2} = \lim_{x \rightarrow 0} \frac{(1+2x)^5 - 1}{2x} \cdot \frac{2}{5} = 5 \cdot \frac{2}{5} = 2$$

441

$$\lim_{x \rightarrow 4} \frac{\ln(x-3)}{x-4} = \frac{0}{0} \quad \text{F.!.}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$y = x - 4$$

$$y \rightarrow 0 \text{ für } x \rightarrow 4$$

$$x - 3 = (y + 4) - 3 = y + 1$$

$$\lim_{x \rightarrow 4} \frac{\ln(x-3)}{x-4} = \lim_{y \rightarrow 0} \frac{\ln(y+1)}{y} = 1$$

442

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x^2 - 3x} =$$

$$= \lim_{x \rightarrow 0} \frac{e^{2x} (e^x - 1)}{x(x-3)} = -\frac{1}{3}$$

The image shows a handwritten solution on grid paper. The original limit is $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x^2 - 3x}$. The numerator is factored as $e^{2x}(e^x - 1)$ and the denominator as $x(x-3)$. The limit is then evaluated as $\lim_{x \rightarrow 0} \frac{e^{2x} (e^x - 1)}{x(x-3)} = -\frac{1}{3}$. Handwritten annotations include a blue arrow pointing to the e^{2x} term with the value '1', a red arrow pointing to the $(e^x - 1)$ term with the value '1', and a blue arrow pointing to the $(x-3)$ term with the value '-3'. The terms e^{2x} and $(e^x - 1)$ are circled in red, and x and $(x-3)$ are circled in pink.