

24/10/2019

424

$$\lim_{x \rightarrow 0} \frac{\ln(x+5) - \ln 5}{x} = \frac{0}{0} \text{ F. !}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\ln\left(\frac{x+5}{5}\right)}{x} = \lim_{x \rightarrow 0} \frac{\ln\left(\frac{x}{5} + 1\right)}{x} = \\ &= \lim_{x \rightarrow 0} \frac{\ln\left(\frac{x}{5} + 1\right)}{\frac{x}{5} \cdot 5} = 1 \cdot \frac{1}{5} = \boxed{\frac{1}{5}} \end{aligned}$$

428

$$\lim_{x \rightarrow 0} \frac{(1+4x^2)^3 - 1}{x^2} = \frac{0}{0} \text{ F. !}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{(1+4x^2)^3 - 1}{4x^2} \cdot 4 = 3 \cdot 4 = \boxed{12} \end{aligned}$$

438

$$\lim_{x \rightarrow -2} \frac{e^{2x+4} - 1}{x+2} = \frac{0}{0} \text{ F. !}$$

$$t = x + 2$$

für  $x \rightarrow -2$  also  $t \rightarrow 0$

$$\begin{aligned} &= \lim_{t \rightarrow 0} \frac{e^{2t} - 1}{t} = \end{aligned}$$

$$\begin{aligned} &= \lim_{t \rightarrow 0} \frac{e^{2t} - 1}{2t} \cdot 2 = 1 \cdot 2 = 2 \end{aligned}$$

**440**  $\lim_{x \rightarrow +\infty} \left( x \ln \frac{3x+1}{3x} \right) = \infty \cdot 0 \quad F.I.$

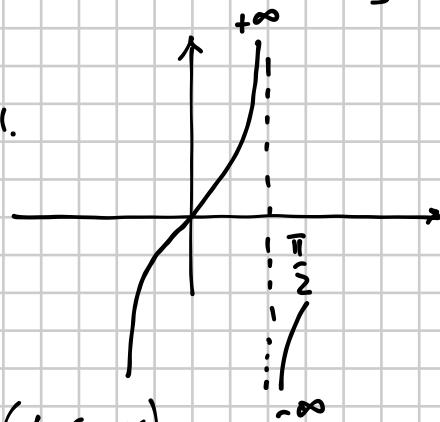
$$= \lim_{x \rightarrow +\infty} \left( x \ln \left( 1 + \frac{1}{3x} \right) \right) =$$

$$= \lim_{y \rightarrow 0^+} \frac{\ln(1+y)}{3y} = \boxed{\frac{1}{3}}$$

$$y = \frac{1}{3x}$$

$y \rightarrow 0$  per  $x \rightarrow +\infty$

$$x = \frac{1}{3y}$$



**447**  $\lim_{x \rightarrow \frac{\pi}{2}^-} (1 - \cos x)^{\tan x} = 1^\infty \quad F.I.$

$$\lim_{x \rightarrow \frac{\pi}{2}^-} (1 - \cos x)^{\tan x} = \lim_{x \rightarrow \frac{\pi}{2}^-} e^{\tan x \cdot \ln(1 - \cos x)}$$

a parte

$$\tan x \cdot \ln(1 - \cos x) = \frac{\sin x}{\cos x} \cdot \ln(1 - \cos x) = \frac{-\sin x}{\cos x} \cdot \ln(1 + (-\cos x))$$

$$\begin{aligned} & \frac{-1}{-\cos x} \cdot \ln(1 + (-\cos x)) \\ & \xrightarrow{x \rightarrow \frac{\pi}{2}^-} -1 \end{aligned}$$

$$\lim_{x \rightarrow \frac{\pi}{2}^-} (1 - \cos x)^{\tan x} = e^{-1} = \boxed{\frac{1}{e}}$$

448

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{8x} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{e^{-x}}(e^{2x} - 1)}{4 \cdot 2x} = \boxed{\frac{1}{4}}$$

↑  
↑  
 $e^{-x}$   
 $(e^{2x} - 1)$

ALTERNATIVA

$$\lim_{x \rightarrow 0} \frac{e^x - 1 + 1 - e^{-x}}{8x} = \lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{8x} - \frac{e^{-x} - 1}{8x} \right] =$$

$$= \lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{8x} + \frac{e^{-x} - 1}{-x \cdot 8} \right] = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$$

↓  
↓  
 $e^x - 1$   
 $e^{-x} - 1$