24/10/2019

424 $\lim_{x \to 0} \frac{\ln(x+5) - \ln 5}{x} = \frac{0}{0}$ F.!

$$= \lim_{x \to 0} \frac{\ln\left(\frac{x+5}{5}\right)}{x} = \lim_{x \to 0} \frac{\ln\left(\frac{x}{5}+1\right)}{x} =$$

$$= \lim_{x \to 0} \frac{\ln\left(\frac{x}{5}+1\right)}{\frac{x}{5} \cdot 5} = 1 \cdot \frac{1}{5} = \boxed{\frac{1}{5}}$$

1

428 $\lim_{x \to 0} \frac{(1+4x^2)^3 - 1}{x^2} = \frac{0}{0}$ F.!

$$= \lim_{x \to 0} \frac{(1+4x^2)^3 - 1}{4x^2} \cdot 4 = 3 \cdot 4 = \boxed{12}$$

3

438 $\lim_{x \to -2} \frac{e^{2x+4} - 1}{x+2} = \frac{0}{0}$ F.! $t = x+2$

per $x \to -2$ si ha $t \to 0$

$$= \lim_{t \to 0} \frac{e^{2t} - 1}{t} =$$

1

$$= \lim_{t \to 0} \frac{e^{2t} - 1}{2t} \cdot 2 = 1 \cdot 2 = 2$$

440 $\displaystyle\lim_{x\to+\infty}\left(x\ln\frac{3x+1}{3x}\right) = \infty\cdot 0$ F.I.

$$= \lim_{x\to+\infty}\left(x\ln\left(1+\frac{1}{3x}\right)\right) =$$

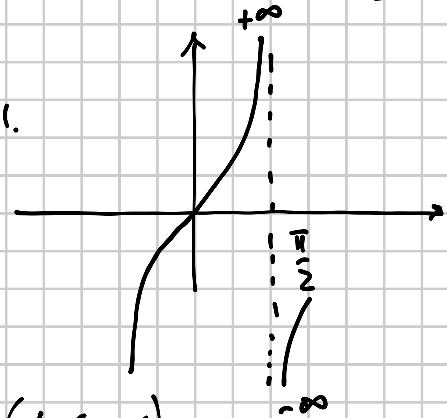
$y = \frac{1}{3x}$

$y \to 0$ per $x\to+\infty$

$x = \frac{1}{3y}$

$$= \lim_{y\to 0} \frac{\overbrace{\ln(1+y)}^{\to 1}}{3y} = \boxed{\frac{1}{3}}$$

447 $\displaystyle\lim_{x\to\frac{\pi}{2}^-}(1-\cos x)^{\tan x} = 1^{\infty}$ F.I.



$$\lim_{x\to\frac{\pi}{2}^-}(1-\cos x)^{\tan x} = \lim_{x\to\frac{\pi}{2}^-} e^{\tan x\cdot\ln(1-\cos x)}$$

a parte

$$\tan x\cdot\ln(1-\cos x) = \frac{\sin x}{\cos x}\cdot\ln(1-\cos x) = \underbrace{\overbrace{-\sin x}^{\to -1}\cdot\overbrace{\frac{\ln(1+(-\cos x))}{-\cos x}}^{\to 1}}_{\substack{x\to\frac{\pi}{2}^- \\ \downarrow \\ -1}}$$

$$\lim_{x\to\frac{\pi}{2}^-}(1-\cos x)^{\tan x} = e^{-1} = \boxed{\frac{1}{e}}$$

448

$$\lim_{x \to 0} \frac{e^x - e^{-x}}{8x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\overbrace{e^{-x}}^{1}\,\overbrace{\frac{(e^{2x}-1)}{2x}}^{1}}{4} = \boxed{\frac{1}{4}}$$

ALTERNATIVA

$$\lim_{x \to 0} \frac{e^x - 1 + 1 - e^{-x}}{8x} = \lim_{x \to 0} \left[\frac{e^x - 1}{8x} - \frac{e^{-x} - 1}{8x} \right] =$$

$$= \lim_{x \to 0} \left[\frac{e^x - 1}{8x} + \frac{e^{-x} - 1}{-x \cdot 8} \right] = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$$