28/11/2019

196 $y = -\dfrac{\sqrt[3]{x^2} + 4x}{\sqrt{x}} = -\dfrac{x^{\frac{2}{3}} + 4x}{x^{\frac{1}{2}}} =$

$$= -x^{\frac{2}{3}-\frac{1}{2}} - 4x^{1-\frac{1}{2}} = -x^{\frac{4-3}{6}} - 4x^{\frac{1}{2}} =$$

$$= -x^{\frac{1}{6}} - 4x^{\frac{1}{2}}$$

$$y' = -\frac{1}{6}x^{\frac{1}{6}-1} - 4 \cdot \frac{1}{2}x^{\frac{1}{2}-1} = -\frac{1}{6}x^{-\frac{5}{6}} - 2x^{-\frac{1}{2}} =$$

$$= -\frac{1}{6} \cdot \frac{1}{x^{\frac{5}{6}}} - 2 \cdot \frac{1}{x^{\frac{1}{2}}} =$$

$$= -\frac{1}{6x^{\frac{5}{6}}} - \frac{2}{x^{\frac{1}{2}}} = -\frac{1 + 12x^{\frac{5}{6}-\frac{1}{2}}}{6x^{\frac{5}{6}}} =$$

$$= -\frac{1 + 12x^{\frac{5-3}{6}}}{6\sqrt[6]{x^5}} = -\frac{1 + 12x^{\frac{1}{3}}}{6\sqrt[6]{x^5}} = -\frac{1 + 12\sqrt[3]{x}}{6\sqrt[6]{x^5}}$$

DERIVATA DEL LOGARITMO

- $y = \ln x$ DOMINIO $= \{ x \in \mathbb{R} \mid x > 0 \}$

$$y' = \lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h} = \lim_{h \to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h} =$$

$$= \lim_{h \to 0} \frac{\ln\left(1 + \frac{h}{x}\right)}{\frac{h}{x} \cdot x} = \frac{1}{x}$$

1

- $f(x) = \log_a x = \frac{\ln x}{\ln a}$ $f'(x) = \overbrace{\frac{1}{\ln a}}^{\text{COSTANTE}} \cdot (\ln x)' = \frac{1}{x \ln a}$

- $y = e^x$ $y' = e^x$

- $f(x) = a^x$ $f(x) = e^{\ln a^x} = e^{x \cdot \ln a}$ ← COMPOSTA

$a > 0 \quad a \neq 1$

$g(x) = e^x$

$h(x) = x \cdot \ln a$

$g(h(x)) = e^{x \cdot \ln a}$

$$\Downarrow$$

$$f'(x) = e^{x \ln a} \cdot \ln a = a^x \cdot \ln a$$

197 $y = \sqrt{\sqrt{x}} - \ln \frac{1}{x^2} + e^4 =$ $\left[y' = \frac{1}{4\sqrt[4]{x^3}} + \frac{2}{x} \right]$

DOMINIO $(x > 0)$

$$= x^{\frac{1}{4}} - \ln x^{-2} + e^4 = x^{\frac{1}{4}} + 2 \ln x + \overbrace{e^4}^{\text{COSTANTE}}$$

$$y' = \frac{1}{4} x^{\frac{1}{4}-1} + 2 \cdot \frac{1}{x} = \frac{1}{4} x^{-\frac{3}{4}} + \frac{2}{x} = \frac{1}{4\sqrt[4]{x^3}} + \frac{2}{x}$$

198 $y = x^2 \ln 4 - x\sqrt{x} + 4 \sin x = x^2 \ln 4 - x^{\frac{3}{2}} + 4 \sin x$

$\left[y' = 2x \ln 4 - \frac{3}{2}\sqrt{x} + 4 \cos x \right]$

$$y' = 2x \cdot \ln 4 - \frac{3}{2} x^{\frac{3}{2}-1} + 4 \cos x =$$

$$= 2x \ln 4 - \frac{3}{2}\sqrt{x} + 4 \cos x$$

DERIVATA DEL PRODOTTO DI FUNZIONI

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

ESEMPIO

$y = \sin x \cdot \ln x \qquad y' = (\sin x)' \cdot \ln x + \sin x \cdot (\ln x)' =$

$$= \cos x \cdot \ln x + \sin x \cdot \frac{1}{x} =$$

$$= \cos x \cdot \ln x + \frac{\sin x}{x}$$

DIMOSTRAZIONE

$$y = f(x) \cdot g(x)$$

$$y' = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x)g(x)}{h} =$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h} =$$

$$= \lim_{h \to 0} \frac{[f(x+h) - f(x)]g(x+h) + f(x)[g(x+h) - g(x)]}{h} =$$

$$= \lim_{h \to 0} \underbrace{\frac{f(x+h) - f(x)}{h}}_{f'(x)} \cdot \underbrace{g(x+h)}_{g(x)} + \lim_{h \to 0} f(x) \cdot \underbrace{\frac{g(x+h) - g(x)}{h}}_{g'(x)} =$$

$$= f'(x)\, g(x) + f(x)\, g'(x)$$

219 $y = \frac{1}{2} e^x (\sin x + \cos x) - e^x \cos x$

$$y' = \frac{1}{2}\left[e^x(\sin x + \cos x) + e^x \cdot (\cos x - \sin x)\right] - \left(e^x \cos x + e^x(-\sin x)\right) =$$

derivata di e^x

derivata di $\sin x + \cos x$

$$= \frac{1}{2}\left[e^x \cancel{\sin x} + e^x \cos x + e^x \cos x - e^x \cancel{\sin x}\right] - e^x \cos x + e^x \sin x =$$

$$= \frac{1}{2} \cdot 2 \cancel{e^x \cos x} - \cancel{e^x \cos x} + e^x \sin x = e^x \sin x$$