3/12/2019

277 $y = \dfrac{2(\tan x - 1)}{\cos x - \sin x}$

$$y' = \frac{2\left[(\tan x - 1)'(\cos x - \sin x) - (\tan x - 1)(-\sin x - \cos x)\right]}{(\cos x - \sin x)^2} =$$

$$= 2\,\frac{\left[\frac{1}{\cos^2 x}(\cos x - \sin x) - \left(\frac{\sin x}{\cos x} - 1\right)(-\sin x - \cos x)\right]}{(\cos x - \sin x)^2} =$$

$$= 2\,\frac{\left[\frac{1}{\cos x} - \frac{\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos x} + \cancel{\sin x} - \cancel{\sin x} - \cos x\right]}{(\cos x - \sin x)^2} =$$

$$= 2\,\frac{\frac{1}{\cos x} - \frac{\sin x}{\cos^2 x} + \frac{1}{\cos x} - \cos x - \cos x}{(\cos x - \sin x)^2} =$$

$$= 2\,\frac{\frac{2}{\cos x} - \frac{\sin x}{\cos^2 x} - 2\cos x}{(\cos x - \sin x)^2} = 2\,\frac{\frac{2\cos x - \sin x - 2\cos^3 x}{\cos^2 x}}{(\cos x - \sin x)^2} =$$

$$= 2\,\frac{2\cos x - \sin x - 2\cos^3 x}{\cos^2 x} \cdot \frac{1}{\underbrace{\cos^2 x + \sin^2 x}_{1} - 2\cos x \sin x} =$$

$$= 2\,\frac{2\cos x\overbrace{(1 - \cos^2 x)}^{\sin^2 x} - \sin x}{\cos^2 x} \cdot \frac{1}{1 - 2\cos x \sin x}$$

$$= 2\,\frac{2\cos x\,\overbrace{(1-\cos^2 x)}^{\sin^2 x} - \sin x}{\cos^2 x}\cdot\frac{1}{1-2\cos x\sin x} =$$

$$= 2\,\frac{2\cos x\sin^2 x - \sin x}{\cos^2 x}\cdot\frac{1}{1-2\cos x\sin x} =$$

$$= 2\,\frac{-\sin x\,\cancel{(1-2\cos x\sin x)}}{\cos^2 x}\cdot\frac{1}{\cancel{1-2\cos x\sin x}} =$$

$$= -\frac{2\sin x}{\cos^2 x}$$

438 $y = \ln(1+\cos^2 x^3)$ $\left[y' = \frac{-6x^2\cos x^3 \sin x^3}{1+\cos^2 x^3}\right]$

$$y' = \frac{1}{1+\cos^2 x^3}\cdot(1+\cos^2 x^3)' = \frac{1}{1+\cos^2 x^3}\cdot\left(2(\cos x^3)\cdot(\cos x^3)'\right) =$$

$$= \frac{2\cos x^3\cdot(-\sin x^3)\cdot 3x^2}{1+\cos^2 x^3} = -\frac{6x^2\cos x^3\cdot\sin x^3}{1+\cos^2 x^3}$$

303 $y = e^{-\sqrt{x}}(2-\sqrt{x})$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$y' = (e^{-\sqrt{x}})'(2-\sqrt{x}) + e^{-\sqrt{x}}\cdot(2-\sqrt{x})' =$$

$$= e^{-\sqrt{x}}\cdot(-\sqrt{x})'(2-\sqrt{x}) + e^{-\sqrt{x}}\cdot\left(-\frac{1}{2\sqrt{x}}\right) =$$

$$= e^{-\sqrt{x}}\cdot\left(-\frac{1}{2\sqrt{x}}\right)(2-\sqrt{x}) + e^{-\sqrt{x}}\cdot\left(-\frac{1}{2\sqrt{x}}\right) = -\frac{e^{-\sqrt{x}}}{2\sqrt{x}}(2-\sqrt{x}+1) = \frac{e^{-\sqrt{x}}(\sqrt{x}-3)}{2\sqrt{x}}$$

DERIVATA DELLA FUNZIONE INVERSA

$f : I \to J$ BIETTIVA F. INVERSA $f^{-1} : J \to I$ tale che $f^{-1}(f(x)) = x \quad \forall x \in I$

CONGETTURA

$f^{-1}(f(x)) = x$ DERIVIAMO AMBO I MEMBRI

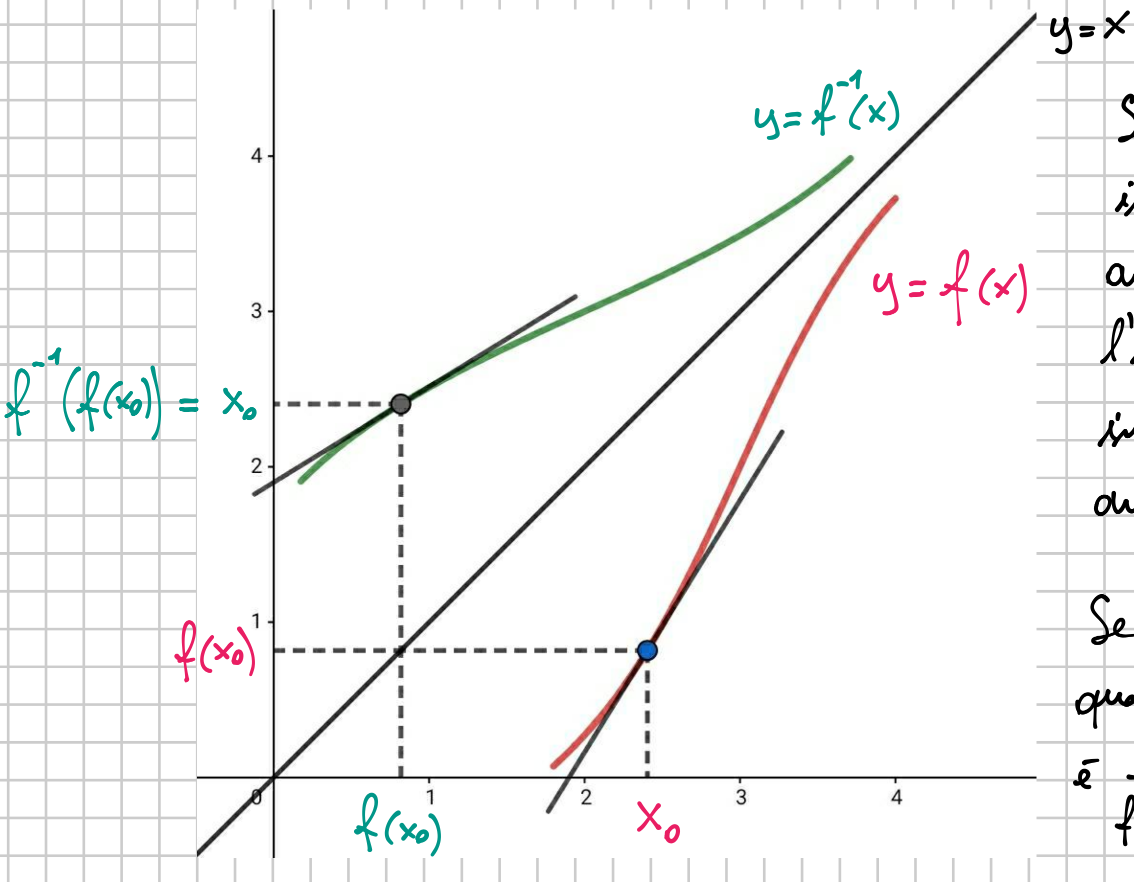
$$(f^{-1})'(f(x)) \cdot f'(x) = 1 \Rightarrow (f^{-1})'(f(x)) = \frac{1}{f'(x)}$$

$y = f(x) \iff f^{-1}(y) = x$

per definizione di inversa

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}$$

$f : I \to \mathbb{R}$ INVERTIBILE E DERIVABILE $\Rightarrow f^{-1}$ È DERIVABILE DOVE $f'(f^{-1}(x)) \neq 0$ E $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$



Se la tangente di f in x_0 ha coeff. angolare $f'(x_0)$, l'inversa ha tangente in $f(x_0)$ con coeff. angolare $\dfrac{1}{f'(x_0)}$.

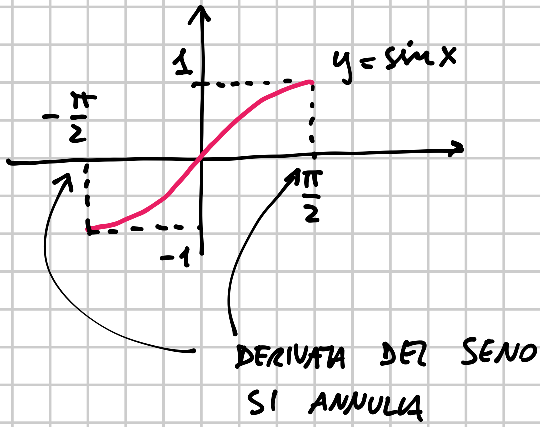
Se chiamo $t = f(x_0)$, questo coeff. angolare è $\dfrac{1}{f'(f^{-1}(t))} = (f^{-1})'(t)$

$$\left(f^{-1}\right)'(x) = \frac{1}{f'\left(f^{-1}(x)\right)}$$

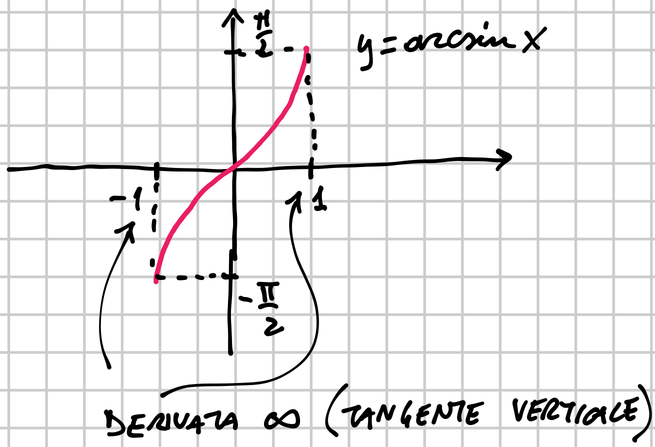
ESEMPI

1) $f(x) = \sin x$

$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to (-1, 1)$



Escludo $-\frac{\pi}{2}$ e $\frac{\pi}{2}$ poiché dove $y = \sin x$ ha derivata nulla, $y = \arcsin x$ (la sua inversa) non è derivabile (ha derivata infinita).



In tutti gli altri punti dell'intervallo $(-1, 1)$ l'arcsin è derivabile e vale la formula

$$(\arcsin x)' = \frac{1}{\sin'(\arcsin x)} = \frac{1}{\cos(\arcsin x)} =$$

$$= \frac{1}{\sqrt{1 - \sin^2(\arcsin x)}} = \frac{1}{\sqrt{1 - x^2}}$$

2) $\arccos : (-1, 1) \to (0, \pi)$

$$(\arccos x)' = \frac{1}{\underbrace{-\sin}_{\cos'}(\arccos x)} = \frac{1}{-\sqrt{1 - \cos^2(\arccos x)}} =$$

$$= -\frac{1}{\sqrt{1 - x^2}}$$

3) arctan : $\mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$(\arctan x)' = \frac{1}{\tan'(\arctan x)} = \frac{1}{1 + \tan^2(\arctan x)} = \frac{1}{1+x^2}$$

4) Calcolare la derivata di $y = \ln x$ come inversa di $y = e^x$

$$(\ln x)' = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

LA NOTAZIONE DI LEIBNIZ $\frac{dy}{dx}$

- **FUNZIONE INVERSA** $\quad \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

ESEMPIO $\quad y = \sqrt[3]{x} \iff x = y^3$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{3y^2} = \frac{1}{3\sqrt[3]{x^2}}$$

- **FUNZIONE COMPOSTA** $\quad \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

ESEMPIO $\quad y = \ln(x^3+1)$

$t = x^3+1 \quad \Longrightarrow \quad y = \ln t \rightarrow \frac{dy}{dt} = \frac{1}{t}$

$t = x^3+1 \rightarrow \frac{dt}{dx} = 3x^2$

$$\frac{dy}{dx} = \underbrace{\frac{1}{t}}_{\frac{dy}{dt}} \cdot \underbrace{3x^2}_{\frac{dt}{dx}} = \frac{3x^2}{x^3+1}$$

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$$y = \ln\sqrt{\frac{1+x}{1-x}}$$

$$y = \ln t$$

$$t = \sqrt{\frac{1+x}{1-x}} = \sqrt{s}$$

$$s = \frac{1+x}{1-x}$$

$$\Downarrow$$

$$\frac{dy}{dt} = \frac{1}{t} \qquad \frac{dt}{ds} = \frac{1}{2\sqrt{s}}$$

$$\frac{ds}{dx} = \frac{1-\cancel{x}+1+\cancel{x}}{(1-x)^2} = \frac{2}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{ds}\cdot\frac{ds}{dx} =$$

$$= \frac{1}{t}\cdot\frac{1}{\cancel{2}\sqrt{s}}\cdot\frac{\cancel{2}}{(1-x)^2} = \frac{1}{\sqrt{s}}\cdot\frac{1}{\sqrt{s}}\cdot\frac{1}{(1-x)^2} =$$

$$= \frac{1}{s\,(1-x)^2} = \frac{1}{\frac{1+x}{\cancel{1-x}}\,(1-x)^{\cancel{2}}} = \frac{1}{1-x^2}$$