14/2/2020

CONCAVITÀ E FLESSI

298 $y = \sqrt{\dfrac{2-x}{x}}$

DOMINIO

$$\frac{2-x}{x} \geq 0 \qquad \begin{array}{lll} 2-x>0 & \Rightarrow & x<2 \\ x>0 & \Rightarrow & x>0 \end{array}$$

		0		2	
$2-x$	+		+	0	−
x	−	X	+		+
	−	X	+	0	−

$D = (0,2]$

$f: (0,2] \to \mathbb{R}$

$$f(x) = \sqrt{\frac{2-x}{x}}$$

$$f'(x) = \frac{1}{2\sqrt{\frac{2-x}{x}}} \cdot \left(\frac{2-x}{x}\right)' = \frac{1}{2}\sqrt{\frac{x}{2-x}} \cdot \frac{-\cancel{x}-2+\cancel{x}}{x^2} = -\frac{1}{x^2}\sqrt{\frac{x}{2-x}}$$

$-x^{-2}$

$x \neq 2$

$$f'(2) = f'_-(2) = \lim_{x\to 2^-} -\frac{1}{x^2}\sqrt{\frac{x}{2-x}} = -\infty$$

$$f''(x) = 2x^{-3}\cdot\sqrt{\frac{x}{2-x}} + \left(-\frac{1}{x^2}\right)\cdot\frac{1}{2\sqrt{\frac{x}{2-x}}}\cdot\frac{2-\cancel{x}+\cancel{x}}{(2-x)^2} =$$

$$= \frac{2}{x^3}\sqrt{\frac{x}{2-x}} - \frac{1}{x^2}\cdot\sqrt{\frac{2-x}{x}}\cdot\frac{1}{(2-x)^2} =$$

$x \neq 0 \quad x \neq 2$

$x \in (0,2)$

$$= \frac{1}{x^2}\sqrt{\frac{x}{2-x}}\left[\frac{2}{x} - \frac{1}{\frac{x}{2-x}}\cdot\frac{1}{(2-x)^2}\right] =$$

$$= \frac{1}{x^2}\sqrt{\frac{x}{2-x}}\left[\frac{2}{x} - \frac{1}{x(2-x)}\right] = \frac{1}{x^3}\sqrt{\frac{x}{2-x}}\left[2 - \frac{1}{2-x}\right]$$

$$f''(x) = \underbrace{\frac{1}{x^3}\sqrt{\frac{x}{2-x}}}_{>0}\left[2 - \frac{1}{2-x}\right] \qquad x \in (0,2)$$

ZERI DI f''

$$2 - \frac{1}{2-x} = 0 \Rightarrow \frac{1}{2-x} = 2 \qquad 1 = 4-2x \qquad x = \frac{3}{2}$$

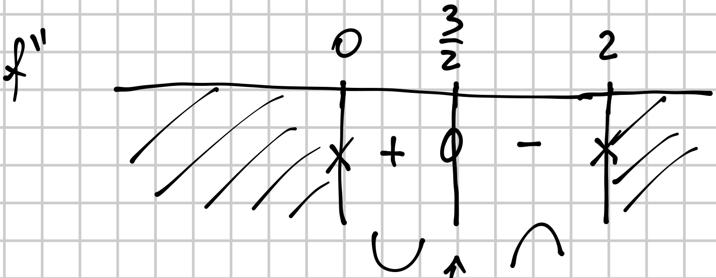
CANDIDATO FLESSO

SEGNO DI f''

$$2 - \frac{1}{2-x} > 0 \qquad \frac{4-2x-1}{\underbrace{2-x}_{>0 \text{ in } (0,2)}} > 0 \Rightarrow 4-2x-1>0$$

$$-2x > -3$$

$$x < \frac{3}{2}$$



$\frac{3}{2}$ è p.to di flesso

TANGENTE INFLESSIONALE: Calcolo $f'\left(\frac{3}{2}\right)$

$$f'(x) = -\frac{1}{x^2}\sqrt{\frac{x}{2-x}}$$

$$f'\left(\frac{3}{2}\right) = -\frac{1}{\frac{9}{4}}\sqrt{\frac{\frac{3}{2}}{2-\frac{3}{2}}} = -\frac{4}{9}\sqrt{\frac{\frac{3}{\cancel{2}}}{\frac{1}{\cancel{2}}}} = -\frac{4}{9}\sqrt{3}$$

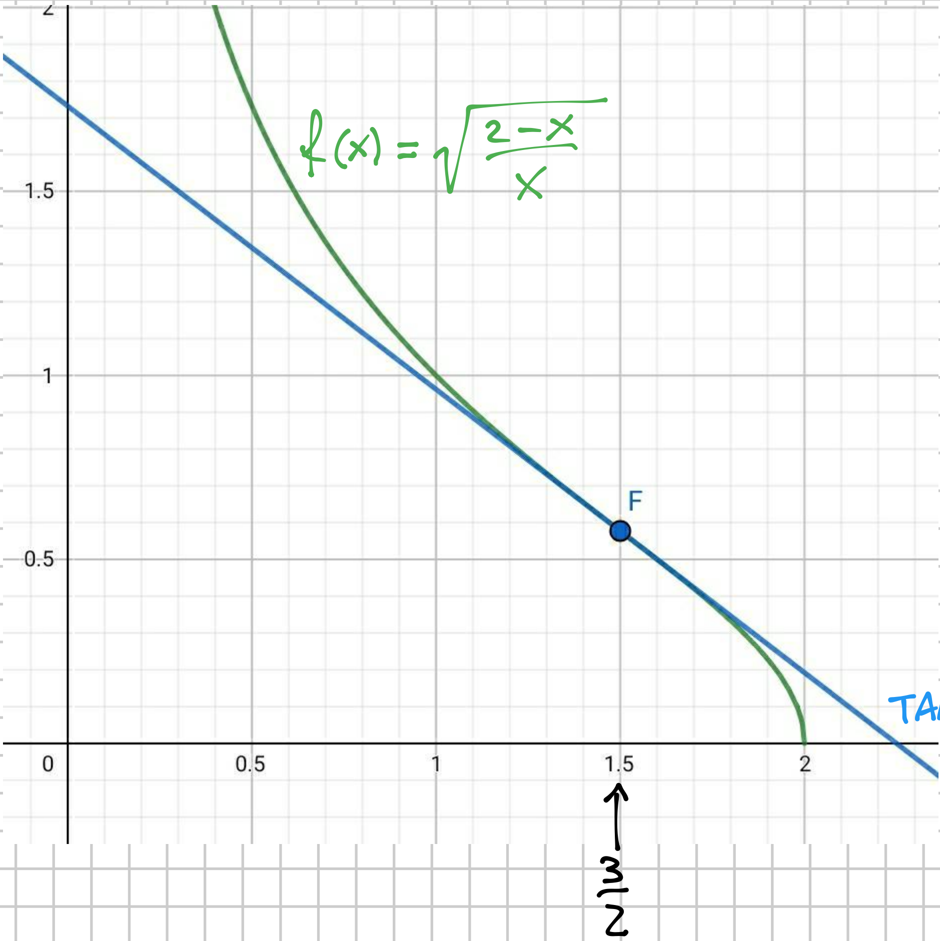
coeff. angolare della tangente

$$f(x) = \sqrt{\frac{2-x}{x}} \qquad f\left(\frac{3}{2}\right) = \sqrt{\frac{2-\frac{3}{2}}{\frac{3}{2}}} = \sqrt{\frac{\frac{1}{\cancel{2}}}{\frac{3}{\cancel{2}}}} = \frac{1}{\sqrt{3}}$$

$$\boxed{y - f(x_0) = f'(x_0)(x-x_0)}$$

EQ. DELLA TANGENTE IN $(x_0, f(x_0))$

$$\boxed{y - \frac{1}{\sqrt{3}} = -\frac{4}{9}\sqrt{3}\left(x - \frac{3}{2}\right)}$$



TANGENTE IN F

$$y - \frac{1}{\sqrt{3}} = -\frac{4}{9}\sqrt{3}\left(x - \frac{3}{2}\right)$$

302 $y = 2e^{-x^2} + 2$ $D = \mathbb{R}$

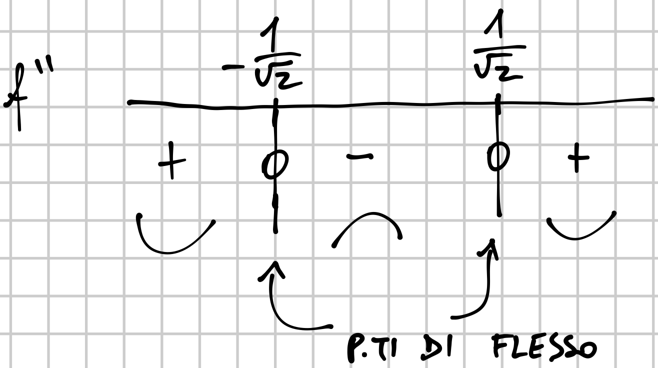
$$f'(x) = 2e^{-x^2} \cdot (-2x) = \underbrace{-4x}_{1°} \cdot \underbrace{e^{-x^2}}_{2°}$$

(0 è p.to stazionario)

$$f''(x) = -4 \cdot e^{-x^2} - 4x \cdot e^{-x^2} \cdot (-2x) = 4e^{-x^2}\left(-1 + 2x^2\right)$$

ZERI DI f'' $-1 + 2x^2 = 0 \implies x = \pm \frac{1}{\sqrt{2}}$

SEGNO DI f'' $2x^2 - 1 > 0 \implies x < -\frac{1}{\sqrt{2}} \vee x > \frac{1}{\sqrt{2}}$



Si può capire che 0 è p.to di massimo