21/10/2020

286 $\sqrt[3]{\frac{1}{2}-\frac{1}{5}}\cdot\sqrt{10}$

287 $\sqrt[3]{\frac{2}{3}-\frac{1}{2}}:\sqrt{\frac{1}{6}}$

288 $\left(\sqrt{6}\cdot\sqrt[3]{\frac{2}{3}-\frac{1}{2}}\right):\sqrt[6]{\frac{1}{36}}$

286

$$\sqrt[3]{\frac{1}{2}-\frac{1}{5}}\cdot\sqrt{10}=\sqrt[3]{\frac{5-2}{10}}\cdot\sqrt{10}=\sqrt[3]{\frac{3}{10}}\cdot\sqrt{10}=$$

$$=\sqrt[6]{\frac{3^2}{10^2}}\cdot\sqrt[6]{10^3}=\sqrt[6]{\frac{3^2}{\cancel{10^2}}\cdot 10^{\cancel{3}}}=\sqrt[6]{9\cdot 10}=\sqrt[6]{90}$$

287

$$\sqrt[3]{\frac{2}{3}-\frac{1}{2}}:\sqrt{\frac{1}{6}}=\sqrt[3]{\frac{4-3}{6}}:\sqrt{\frac{1}{6}}=\sqrt[3]{\frac{1}{6}}:\sqrt{\frac{1}{6}}=$$

$$=\sqrt[6]{\frac{1}{6^2}}:\sqrt[6]{\frac{1}{6^3}}=\sqrt[6]{\frac{1}{6^2}:\frac{1}{6^3}}=\sqrt[6]{\frac{1}{\cancel{6^2}}\cdot 6^{\cancel{3}}}=\sqrt[6]{6}$$

288

$$\left(\sqrt{6}\cdot\sqrt[3]{\frac{2}{3}-\frac{1}{2}}\right):\sqrt[6]{\frac{1}{36}}=$$

$$=\sqrt{6}\cdot\sqrt[3]{\frac{4-3}{6}}:\sqrt[6]{\frac{1}{6^2}}=\sqrt{6}\cdot\sqrt[3]{\frac{1}{6}}:\sqrt[6]{\frac{1}{6^2}}=$$

$$=\sqrt[6]{6^3\cdot\frac{1}{\cancel{6^2}}:\frac{1}{\cancel{6^2}}}=\sqrt[\cancel{6}\,2]{6^{\cancel{3}}}=\sqrt{6}$$

POTENZE DI RADICI

322 $\left(\sqrt[4]{3}\right)^3 = \sqrt[4]{3^3} = \sqrt[4]{27}$

323 $\left(\sqrt[3]{2}\right)^2 = \sqrt[3]{2^2} = \sqrt[3]{4}$

324 $\left(\sqrt[5]{9}\right)^2 = \sqrt[5]{9^2} = \sqrt[5]{81}$

ALTRI ESEMPI

$\left(\sqrt{2}\right)^2 = 2$ infatti $\left(\sqrt{2}\right)^2 = \sqrt{2^2} = \sqrt{4} = 2$

$\left(\sqrt[3]{7}\right)^2 = \sqrt[3]{7^2} = \sqrt[3]{49}$

$\left(\sqrt[3]{7}\right)^2 = \sqrt[3]{7} \cdot \sqrt[3]{7} = \sqrt[3]{7 \cdot 7} = \sqrt[3]{7^2}$

RADICI DI UN RADICALE

332 $\sqrt[3]{\sqrt{3}} = \sqrt[6]{3}$ $\longrightarrow$ infatti $\left(\sqrt[3]{\sqrt{3}}\right)^6 = \cancel{\sqrt[3]{}}\left(\sqrt{3}\right)^{\cancel{6}\,2} =$

$= \left(\sqrt{3}\right)^2 = 3$

$\Downarrow$

quindi $\sqrt[3]{\sqrt{3}}$ elevato a 6 dà 3, cioè $\sqrt[3]{\sqrt{3}} = \sqrt[6]{3}$

333 $\sqrt{\sqrt{2}} = \sqrt[4]{2}$

334 $\sqrt{\sqrt[3]{2}} = \sqrt[6]{2}$

335 $\sqrt{\sqrt{\sqrt[3]{3}}} = \sqrt[12]{3}$

TEOREMA 5 | Alcune operazioni tra radicali

Nell'ipotesi che siano verificate le condizioni di esistenza di tutti i radicali al primo membro, valgono le seguenti proprietà:

a. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ con $n \in \mathbf{N} - \{0\}$

b. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ con $n \in \mathbf{N} - \{0\}$

c. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$ con $n, m \in \mathbf{N} - \{0\}$

d. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$ con $n, m \in \mathbf{N} - \{0\}$

TRASPORTO SOTTO IL SEGNO DI RADICE

346 $2\sqrt{2}$; $\quad -3\sqrt{3}$ $\quad [\sqrt{8}; -\sqrt{27}]$

$$2\sqrt{2} = \sqrt{2^2} \cdot \sqrt{2} = \sqrt{2^2 \cdot 2} = \sqrt{2^3} = \sqrt{8}$$

OBIETTIVO = scrivere $2\sqrt{2}$ come radicale del tipo $\sqrt{a}$

$$5\sqrt{3} = \underbrace{\sqrt{5^2} \cdot \sqrt{3}}_{\text{questo passaggio lo saltiamo}} = \sqrt{5^2 \cdot 3} = \sqrt{75}$$

$$-3\sqrt{3} = -\sqrt{3^2 \cdot 3} = -\sqrt{27}$$

IN GENERALE

$$a\sqrt[m]{b} = \sqrt[m]{a^m \cdot b}$$

$a, b > 0$

347 $3\sqrt{2}$; $\quad -2\sqrt{3}$ $\quad [\sqrt{18}; -\sqrt{12}]$

348 $\frac{1}{2}\sqrt{8}$; $\quad -\frac{2}{3}\sqrt{\frac{3}{2}}$ $\quad \left[\sqrt{2}; -\sqrt{\frac{2}{3}}\right]$

$$3\sqrt{2} = \sqrt{3^2 \cdot 2} = \sqrt{18} \qquad -2\sqrt{3} = -\sqrt{2^2 \cdot 3} = -\sqrt{12}$$

$$\frac{1}{2}\sqrt{8} = \sqrt{\frac{1}{2^2} \cdot 8} = \sqrt{\frac{1}{4} \cdot 8} = \sqrt{2}$$

$$-\frac{2}{3}\sqrt{\frac{3}{2}} = -\sqrt{\frac{2^{\cancel{2}}}{3^{\cancel{2}}} \cdot \frac{\cancel{3}}{\cancel{2}}} = -\sqrt{\frac{2}{3}}$$

TRASPORTO FUORI DAL SEGNO DI RADICE

Trasporta tutti i fattori possibili fuori dal segno di radice, ~~supponendo che tutte le variabili rappresentino numeri non negativi.~~

403 $\sqrt{12}$; $\quad \sqrt{50}$ $\quad [2\sqrt{3}; 5\sqrt{2}]$

404 $\sqrt{27}$; $\quad \sqrt{18}$ $\quad [3\sqrt{3}; 3\sqrt{2}]$

405 $\sqrt{200}$; $\quad \sqrt{63}$ $\quad [10\sqrt{2}; 3\sqrt{7}]$

$$\sqrt{12} = \sqrt{2^2 \cdot 3} = 2\sqrt{3} \qquad \sqrt{50} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$$

LO DIVIDO PER L'INDICE DELLA RADICE

$$\sqrt{27} = \sqrt{3^3} = 3\sqrt{3} \qquad \sqrt{18} = \sqrt{3^2 \cdot 2} = 3\sqrt{2}$$

DIVIDO PER 2 :
QUOZIENTE = 1
RESTO = 1

$$\sqrt{200} = \sqrt{5^2 \cdot 2^3} = 5^1 \cdot 2^1 \sqrt{2^1} = 10\sqrt{2}$$

siccome gli esponenti sono $\geq$ dell'indice, entrambi i fattori 5 e 2 possono essere trasportati fuori

$$\sqrt{10^2 \cdot 2} = 10\sqrt{2}$$

$$\sqrt{63} = \sqrt{3^2 \cdot 7} = 3\sqrt{7}$$

$$\sqrt[3]{2^8 \cdot 5^2 \cdot 7^6 \cdot 11^{16}} = 2^2 \cdot 7^2 \cdot 11^5 \cdot \sqrt{2^2 \cdot 5^2 \cdot 11^1}$$

2^8: porto fuori

5^2: NO perché $2<3$

$7^6 \cdot 11^{16}$: porto fuori

[redacted]

411 $\sqrt{2^8 \cdot 5^5 - 2^{14} \cdot 5^2} =$ $[80\sqrt{61}]$

$$= \sqrt{2^8 \cdot 5^2 \cdot (5^3 - 2^6)} =$$

$$x^8 y^5 - x^{14} y^2 =$$
$$= x^8 y^2 (y^3 - x^6)$$

$$= 2^4 \cdot 5 \sqrt{5^3 - 2^6} =$$

$$= 80 \sqrt{125 - 64} = 80\sqrt{61}$$

413 $\sqrt{24^2 - 192}$; $\sqrt{21^2 + 810}$ $[8\sqrt{6}; 3\sqrt{139}]$

$$\sqrt{24^2 - 192} = \sqrt{(3 \cdot 2^3)^2 - 3 \cdot 2^6} =$$

192	2
96	2
48	2
24	$3 \cdot 2^3$
1	

$192 = 3 \cdot 2^6$

$$= \sqrt{3^2 \cdot 2^6 - 3 \cdot 2^6} = \sqrt{3 \cdot 2^6 (3 - 1)} =$$

$$= \sqrt{3 \cdot 2^6 \cdot 2} = \sqrt{3 \cdot 2^7} = 2^3 \cdot \sqrt{3 \cdot 2}$$

$$= 8\sqrt{6}$$

$$\sqrt{21^2 + 810} = \sqrt{3^2 \cdot 7^2 + 2 \cdot 3^4 \cdot 5} = \sqrt{3^2 (7^2 + 2 \cdot 3^2 \cdot 5)} =$$

$$= 3\sqrt{49 + 90} = 3\sqrt{139}$$

420 $\sqrt[3]{5^3 \cdot 7^4}$; $\sqrt[3]{2^4 \cdot 5^6 \cdot 7^3}$ $[35\sqrt[3]{7}; 350\sqrt[3]{2}]$

$$\sqrt[3]{5^3 \cdot 7^4} = 5 \cdot 7 \sqrt[3]{7} = 35\sqrt[3]{7}$$

$$\sqrt[3]{2^4 \cdot 5^6 \cdot 7^3} = 2 \cdot 5^2 \cdot 7 \sqrt{2} = 350\sqrt{2}$$

SOMME DI RADICALI

459 $2\sqrt{2} + \sqrt{3} - 3\sqrt{2} - 4\sqrt{3} + \sqrt{2}$

$$\cancel{2a} + b - \cancel{3a} - 4b + \cancel{a} = -3b$$

DEVO SOMMARE I RADICALI SIMILI

$$\cancel{2\sqrt{2}} + \underset{0}{\sqrt{3}} - \cancel{3\sqrt{2}} - \underset{0}{4\sqrt{3}} + \cancel{\sqrt{2}} = -3\sqrt{3}$$

460 $\sqrt{8}+\sqrt{2}+\sqrt{27}+\sqrt{12} =$

$= \sqrt{2^3} + \sqrt{2} + \sqrt{3^3} + \sqrt{2^2 \cdot 3} =$

$= 2\sqrt{2} + \sqrt{2} + 3\sqrt{3} + 2\sqrt{3} =$

$= 3\sqrt{2} + 5\sqrt{3}$

465 $\sqrt{18}+\sqrt{12}+\sqrt[6]{27}+\sqrt{2} =$ $[4\sqrt{2}+3\sqrt{3}]$

$= \sqrt{3^2 \cdot 2} + \sqrt{2^2 \cdot 3} + \sqrt[\cancel{6}\,2]{3^{\cancel{3}}} + \sqrt{2} =$

$= 3\sqrt{2} + 2\sqrt{3} + \sqrt{3} + \sqrt{2} = 4\sqrt{2} + 3\sqrt{3}$

507 $(\sqrt{18}+\sqrt{50}) : \sqrt{2} + (\sqrt{5}+1)^2 + (\sqrt{5}-1)(\sqrt{5}+1) =$

$= \left(\sqrt{3^2 \cdot 2} + \sqrt{5^2 \cdot 2}\right) : \sqrt{2} + \underset{(\sqrt{5})^2}{5} + \underset{2 \cdot \sqrt{5} \cdot 1}{2\sqrt{5}} + \underset{1^2}{\cancel{1}} + 5 - \cancel{1} =$

$= (3\sqrt{2} + 5\sqrt{2}) : \sqrt{2} + 10 + 2\sqrt{5} =$

$= \underbrace{8\sqrt{2} : \sqrt{2}}_{\frac{8\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}} + 10 + 2\sqrt{5} = 8 + 10 + 2\sqrt{5} = \boxed{18 + 2\sqrt{5}}$

512 $(\sqrt{3}-\sqrt{6})^2-(1+2\sqrt{2})^2+(1-\sqrt{18})(1+\sqrt{8})+\sqrt{800}+11$

$$3+6-2\sqrt{3}\cdot\sqrt{6}-\left(1+(2\sqrt{2})^2+4\sqrt{2}\right)+1+\sqrt{8}-\sqrt{18}-\sqrt{18\cdot 8}$$

$$+\sqrt{800}+11=$$

$$2\cdot\sqrt{3^2\cdot 2}$$

$$=9-2\sqrt{18}-\left(1+4\cdot 2+4\sqrt{2}\right)+1+\sqrt{2^3}-\sqrt{3^2\cdot 2}-\cancel{\sqrt{3^{\cancel{2}\,1}\cdot 2^{\cancel{4}\,2}}}+$$

$$+\sqrt{2^5\cdot 5^2}+11=$$

$$=\cancel{9}-2\cdot 3\sqrt{2}\cancel{-1}\cancel{-8}-4\sqrt{2}\cancel{+1}+2\sqrt{2}-3\sqrt{2}\cancel{-12}+$$

$$+2^2\cdot 5\cdot\sqrt{2}\cancel{+11}=$$

$$=-6\sqrt{2}-4\sqrt{2}+2\sqrt{2}-3\sqrt{2}+20\sqrt{2}=\boxed{9\sqrt{2}}$$