23/10/2020

670 $\dfrac{x}{\sqrt{5}+1}+\dfrac{1}{\sqrt{5}-1}=x(\sqrt{5}+1)$ $\left[\dfrac{5-\sqrt{5}}{10}\right]$

$$\frac{x(\sqrt5-1)+(\sqrt5+1)}{\cancel{(\sqrt5+1)(\sqrt5-1)}} = \frac{x(\sqrt5+1)\overbrace{(\sqrt5+1)(\sqrt5-1)}^{5-1}}{\cancel{(\sqrt5+1)(\sqrt5-1)}}$$

$$\sqrt5\,x - x + \sqrt5 + 1 = 4\sqrt5\,x + 4x$$

$$\sqrt5\,x - x - 4\sqrt5\,x - 4x = -\sqrt5 - 1$$

$$-3\sqrt5\,x - 5x = -\sqrt5 - 1$$

CAMBIO SEGNI

$$3\sqrt5\,x + 5x = \sqrt5 + 1$$

$$x(3\sqrt5+5) = \sqrt5 + 1$$

$$x = \frac{\sqrt5+1}{3\sqrt5+5}\cdot\frac{3\sqrt5-5}{3\sqrt5-5} =$$

$$= \frac{15-5\sqrt5+3\sqrt5-5}{9\cdot5-25} = \frac{10-2\sqrt5}{20} =$$

$$= \frac{\cancel{2}(5-\sqrt5)}{\cancel{20}_{10}} = \boxed{\frac{5-\sqrt5}{10}}$$

665 $x(x-2\sqrt{2})=(x-\sqrt{2})(x+2\sqrt{2})$

$$\cancel{x^2}-2\sqrt{2}\,x=\cancel{x^2}+2\sqrt{2}\,x-\sqrt{2}\,x-4$$

$$-2\sqrt{2}\,x-2\sqrt{2}\,x+\sqrt{2}\,x=-4$$

$$-3\sqrt{2}\,x=-4$$

$$x=\frac{4}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\overset{2}{\cancel{4}}\sqrt{2}}{\underset{3}{\cancel{6}}}=\boxed{\frac{2\sqrt{2}}{3}}$$

667 $\left(\frac{x}{\sqrt{2}}-\sqrt{2}\right)^2-\frac{1}{2}(x-\sqrt{3})^2=-\frac{1}{2}$

$$\frac{x^2}{2}+2-2x-\frac{1}{2}\left(x^2+3-2\sqrt{3}\,x\right)=-\frac{1}{2}$$

$$2\cdot\frac{x}{\sqrt{2}}\cdot(-\sqrt{2})$$

$$\frac{\cancel{x^2}}{\cancel{2}}+2-2x-\frac{1}{\cancel{2}}\cancel{x^2}-\frac{3}{2}+\sqrt{3}\,x=-\frac{1}{2}$$

$$-2x+\sqrt{3}\,x=-\frac{1}{2}-2+\frac{3}{2}$$

$$-\frac{1}{2}+\frac{3}{2}=\frac{2}{2}=1$$

$$-2x+\sqrt{3}\,x=1-2 \Rightarrow 2x-\sqrt{3}\,x=1$$

$$x\left(2-\sqrt{3}\right)=1 \qquad x=\frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{4-3}=\boxed{2+\sqrt{3}}$$

674 $$\frac{1}{x^2-2}+\frac{1}{x^2-2x\sqrt{2}+2}=\frac{2}{x^2+2x\sqrt{2}+2}$$

$$(x+\sqrt{2})(x-\sqrt{2}) \qquad (x-\sqrt{2})^2 \qquad (x+\sqrt{2})^2$$

C.E.

$x \neq \pm\sqrt{2}$

$$\frac{(x+\sqrt{2})(x-\sqrt{2})+(x+\sqrt{2})^2}{\cancel{(x+\sqrt{2})^2(x-\sqrt{2})^2}}=\frac{2(x-\sqrt{2})^2}{\cancel{(x+\sqrt{2})^2(x-\sqrt{2})^2}}$$

$$x^2\cancel{-2}+x^2+2\sqrt{2}x\cancel{+2}=2(x^2-2\sqrt{2}x+2)$$

$$\cancel{2x^2}+2\sqrt{2}x=\cancel{2x^2}-4\sqrt{2}x+4$$

$$2\sqrt{2}x+4\sqrt{2}x=4$$

$$6\sqrt{2}x=4$$

$$x=\frac{4}{6\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{4\sqrt{2}}{\cancel{12}_{3}}=\boxed{\frac{\sqrt{2}}{3}}$$

dopo il controllo C.E.

675 $\dfrac{x^3+x^2-2}{x^3-2}-\dfrac{1}{x-\sqrt[3]{2}}=1$

C.E.
$x \neq \sqrt[3]{2}$

$$x^3-2 = x^3-(\sqrt[3]{2})^3 = (x-\sqrt[3]{2})\underbrace{(x^2+\sqrt[3]{2}\,x+\sqrt[3]{4})}$$

$A^3-B^3 = (A-B)(A^2+AB+B^2)$

$A^3+B^3 = (A+B)(A^2-AB+B^2)$

IL FALSO QUADRATO DI 2° GRADO
NON È ULTERIORMENTE SCOMPONIBILE

$$\frac{x^3+x^2-2}{(x-\sqrt[3]{2})(x^2+\sqrt[3]{2}\,x+\sqrt[3]{4})}-\frac{1}{x-\sqrt[3]{2}}=1$$

$$\frac{x^3+x^2-2-(x^2+\sqrt[3]{2}\,x+\sqrt[3]{4})}{\cancel{(x-\sqrt[3]{2})(x^2+\sqrt[3]{2}\,x+\sqrt[3]{4})}}=\frac{x^3-2}{\cancel{(x-\sqrt[3]{2})(x^2+\sqrt[3]{2}\,x+\sqrt[3]{4})}}$$

$$\cancel{x^3}+\cancel{x^2}-\cancel{2}-\cancel{x^2}-\sqrt[3]{2}\,x-\sqrt[3]{4}=\cancel{x^3}-\cancel{2}$$

$$-\sqrt[3]{2}\,x=\sqrt[3]{4}$$

$$x=-\frac{\sqrt[3]{4}}{\sqrt[3]{2}}=-\sqrt[3]{\frac{4}{2}}=\boxed{-\sqrt[3]{2}}$$

dopo controllo C.E.

676 $$\frac{1}{x^3-3x^2\sqrt{2}+6x-2\sqrt{2}}=\frac{x}{x^2-2x\sqrt{2}+2}+\frac{1}{\sqrt{2}-x}$$

$(x-\sqrt{2})^3$ $\qquad$ $(x-\sqrt{2})^2$ $\qquad$ $-(x-\sqrt{2})$

C.E.

$x \neq \sqrt{2}$

$$\frac{1}{\cancel{(x-\sqrt{2})^3}}=\frac{x(x-\sqrt{2})-(x-\sqrt{2})^2}{\cancel{(x-\sqrt{2})^3}}$$

$$1 = x^2-\sqrt{2}x-(x^2-2\sqrt{2}x+2)$$

$$1 = \cancel{x^2}-\sqrt{2}x-\cancel{x^2}+2\sqrt{2}x-2$$

$$\sqrt{2}x-2\sqrt{2}x=-2-1$$

$$-\sqrt{2}x=-3$$

$$\sqrt{2}x=3$$

$$x=\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\boxed{\frac{3\sqrt{2}}{2}}$$

dopo controllo C.E.

588 $\begin{cases} x\sqrt{2} - 4y = 2 \\ x - \sqrt{2}\,y = -\sqrt{2} \end{cases}$ $\qquad [(-3\sqrt{2}, -2)]$

$$\begin{cases} (\sqrt{2}\,y - \sqrt{2})\cdot\sqrt{2} - 4y = 2 \\ x = \sqrt{2}\,y - \sqrt{2} \end{cases} \qquad \begin{cases} 2y - 2 - 4y = 2 \\ // \end{cases}$$

SOSTITUZIONE

$$\begin{cases} -2y = 4 \\ // \end{cases} \qquad \begin{cases} y = -2 \\ x = \sqrt{2}\cdot(-2) - \sqrt{2} = \end{cases}$$

$$= -2\sqrt{2} - \sqrt{2} = -3\sqrt{2}$$

$$\boxed{\begin{cases} x = -3\sqrt{2} \\ y = -2 \end{cases}}$$

589 $\begin{cases} x\sqrt{3} - y\sqrt{2} = -\sqrt{6} \\ x\sqrt{2} - y\sqrt{3} = -4 \end{cases}$

$$D = \begin{vmatrix} \sqrt{3} & -\sqrt{2} \\ \sqrt{2} & -\sqrt{3} \end{vmatrix} = -3 + 2 = -1$$

$$D_y = \begin{vmatrix} \sqrt{3} & -\sqrt{6} \\ \sqrt{2} & -4 \end{vmatrix} = -4\sqrt{3} + \sqrt{12} = -4\sqrt{3} + 2\sqrt{3} = -2\sqrt{3}$$

$$D_x = \begin{vmatrix} -\sqrt{6} & -\sqrt{2} \\ -4 & -\sqrt{3} \end{vmatrix} = \sqrt{18} - 4\sqrt{2} = 3\sqrt{2} - 4\sqrt{2} = -\sqrt{2}$$

$$x = \frac{D_x}{D} = \frac{-\sqrt{2}}{-1} = \sqrt{2} \qquad y = \frac{D_y}{D} = \frac{-2\sqrt{3}}{-1} = 2\sqrt{3}$$

POTENZE A ESPONENTE RAZIONALE

$a \geq 0$ BASE

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

$n, m \in \mathbb{N}$

$n, m \geq 1$

$$3^{\frac{1}{2}} = \sqrt{3} \qquad 7^{\frac{5}{8}} = \sqrt[8]{7^5}$$

$$27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$$

Perché è conveniente che $3^{\frac{1}{2}} = \sqrt{3}$?

$$\left(3^{\frac{1}{2}}\right)^2 = 3^{\frac{1}{2}\cdot 2} = 3^1 = 3$$

quindi $3^{1/2}$ elevato al quadrato deve dare 3. Ma allora $3^{1/2} = \sqrt{3}$

591 $\begin{cases} 27^{-\frac{1}{3}}x + 16^{-\frac{1}{2}}y = 36^{-\frac{1}{2}} \\ x - 8^{\frac{2}{3}}y = 10 \end{cases}$ $[(2, -2)]$

$$\begin{cases} \frac{1}{27^{1/3}}x + \frac{1}{16^{1/2}}y = \frac{1}{36^{1/2}} \\ x - \sqrt[3]{8^2}\,y = 10 \end{cases}$$

$$\left(2^3\right)^{\frac{2}{3}} = 2^{\cancel{3}\cdot\frac{2}{\cancel{3}}} = 2^2 = 4$$

$$\begin{cases} \frac{1}{\sqrt[3]{3^3}}x + \frac{1}{\sqrt{16}}y = \frac{1}{\sqrt{36}} \\ x - \sqrt[\cancel{3}]{2^{\cancel{6}\,2}}\,y = 10 \end{cases}$$

$$\begin{cases} \frac{1}{3}x + \frac{1}{4}y = \frac{1}{6} \\ x - 4y = 10 \end{cases}$$

$$\begin{cases} \frac{1}{3}(4y+10) + \frac{1}{4}y = \frac{1}{6} \\ x = 4y + 10 \end{cases}$$

$$\begin{cases} \frac{1}{3}(4y+10)+\frac{1}{4}y=\frac{1}{6} \\ x=4y+10 \end{cases} \qquad \begin{cases} \frac{4}{3}y+\frac{10}{3}+\frac{1}{4}y=\frac{1}{6} \\ // \end{cases}$$

$$\begin{cases} \frac{16y+40+3y}{\cancel{12}}=\frac{2}{\cancel{12}} \\ // \end{cases} \qquad \begin{cases} 19y=-38 \\ // \end{cases} \qquad \begin{cases} y=-2 \\ x=-8+10 \\ \quad =2 \end{cases}$$

$$\boxed{\begin{cases} x=2 \\ y=-2 \end{cases}}$$