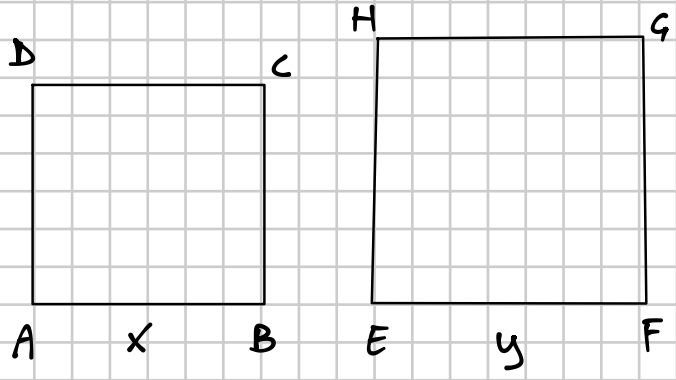
6/11/2020

687 Le misure dei lati di due quadrati differiscono di 3 cm. La somma delle loro aree è 29 cm^2. Determina le misure dei lati dei due quadrati. [2 cm, 5 cm]



$$\begin{cases} x = y - 3 \\ x^2 + y^2 = 29 \end{cases}$$

$$(y-3)^2 + y^2 = 29$$

$$y^2 + 9 - 6y + y^2 = 29$$

$$2y^2 - 6y + 9 - 29 = 0$$

$$2y^2 - 6y - 20 = 0$$

↓ divido per 2 tutti i coefficienti

$$y^2 - 3y - 10 = 0$$

$$\Delta = (-3)^2 - 4 \cdot 1 \cdot (-10) = 49$$

$$y = \frac{3 \pm \sqrt{49}}{2} = \frac{3 \pm 7}{2} = \begin{cases} -2 & \text{N.A. (per il problema)} \\ 5 \end{cases}$$

$$\begin{cases} x = 5 - 3 = 2 \\ y = 5 \end{cases}$$

$AB = 2$ cm $EF = 5$ cm

OSSERVAZIONE

$y^2 - 3y - 10 = 0 \Rightarrow (y-5)(y+2) = 0$

$y - 5 = 0 \Rightarrow y = 5$ ✓

$y + 2 = 0 \Rightarrow y = -2$

$ax^2+bx+c=0$ $\quad \Delta<0 \Rightarrow$ IMP. IN $\mathbb{R}$

$a \neq 0$ $\quad \Delta \geq 0 \Rightarrow$ 2 SOLUZIONI

$\Delta = b^2-4ac$ $\quad x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

(NEL CASO $\Delta=0$, SI HANNO 2 SOL. COINCIDENTI:

$x = -\dfrac{b}{2a}$)

FORMULA RIDOTTA

SI USA IN UN'EQ. COMPLETA

$ax^2+bx+c=0$ CON b PARI (o contiene il fattore 2)

$b=2\beta$ $\quad ax^2+2\beta x+c=0$

$\Delta = b^2-4ac = (2\beta)^2-4ac = 4\beta^2-4ac = 4(\beta^2-ac)$

$\Delta \geq 0$ $\quad x = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{-2\beta \pm \sqrt{4(\beta^2-ac)}}{2a} =$

$= \dfrac{-2\beta \pm 2\sqrt{\beta^2-ac}}{2a} = \dfrac{\cancel{2}(-\beta \pm \sqrt{\beta^2-ac})}{\cancel{2}a} =$

$= \dfrac{-\beta \pm \sqrt{\beta^2-ac}}{a}$

ESEMPIO DI USO DI FORMULA RIDOTTA

149 $4x^2 - 4x - 3 = 0$ $\left[-\frac{1}{2}; \frac{3}{2}\right]$

$b = -4 \qquad \beta = -2$

$\underbrace{\beta^2 - ac}_{\frac{\Delta}{4}} = (-2)^2 - 4 \cdot (-3) = 4 + 12 = 16$

$$x = \frac{-\beta \pm \sqrt{\beta^2 - ac}}{a}$$

$$x = \frac{2 \pm \sqrt{16}}{4} = \frac{2 \pm 4}{4} = \begin{cases} -\frac{2}{4} = -\frac{1}{2} \\ \frac{6}{4} = \frac{3}{2} \end{cases}$$

$$\boxed{x = -\frac{1}{2} \quad \vee \quad x = \frac{3}{2}}$$

CON LA FORMULA SOLITA:

$\Delta = (-4)^2 - 4 \cdot 4 \cdot (-3) = 16 + 48 = 64$

$$x = \frac{4 \pm \sqrt{64}}{8} = \frac{4 \pm 8}{8} = \begin{cases} -\frac{4}{8} = -\frac{1}{2} \\ \frac{12}{8} = \frac{3}{2} \end{cases}$$

153 $x^2 - 2\sqrt{3}x + 2 = 0$ $[\sqrt{3} \pm 1]$

$\beta = -\sqrt{3}$ $\quad \frac{\Delta}{4} = \beta^2 - ac = (-\sqrt{3})^2 - 2 = 3 - 2 = 1$

$$x = \frac{-\beta \pm \sqrt{\beta^2 - ac}}{a} = \sqrt{3} \pm 1$$

151 $4x^2 - 60x - 31 = 0$ $\left[-\frac{1}{2}; \frac{31}{2}\right]$

$\beta = -30$ $\quad \frac{\Delta}{4} = \beta^2 - ac = (-30)^2 - 4(-31) = 900 + 124 = \underbrace{1024}_{32^2}$

$$x = \frac{30 \pm 32}{4} = \begin{cases} -\frac{2}{4} = -\frac{1}{2} \\ \frac{62}{4} = \frac{31}{2} \end{cases}$$

$\boxed{x = -\frac{1}{2} \vee x = \frac{31}{2}}$

$\Delta > 0 \iff \frac{\Delta}{4} = \beta^2 - ac > 0$ 2 SOLUZ. REALI DISTINTE

$\Delta = 0 \iff \frac{\Delta}{4} = \beta^2 - ac = 0$ 2 SOLUZ. REALI COINCIDENTI

$\Delta < 0 \iff \frac{\Delta}{4} = \beta^2 - ac < 0$ NESSUNA SOLUZIONE REALE