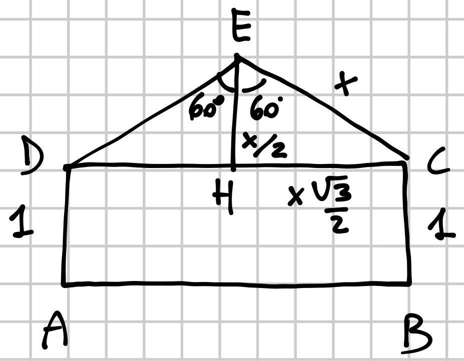
2/12/2020

144 Sia $ABCD$ un rettangolo in cui $BC = AD = 1$ cm. Costruisci, esternamente al rettangolo, il triangolo CED, isoscele sulla base CD, avente l'angolo $C\hat{E}D$ di ampiezza 120°. Sapendo che l'area del pentagono $ABCED$ è $8\sqrt{3}\ \text{cm}^2$, determina il perimetro del pentagono. $[(10 + 4\sqrt{3})\ \text{cm}]$



$\overline{CE} = x \qquad \overline{HC} = \frac{\sqrt{3}}{2}x$

$\overline{EH} = \frac{x}{2}$

$\mathcal{A}_{ABCDE} = \mathcal{A}_{ABDC} + 2\,\mathcal{A}_{EHC}$

$$\underbrace{x\sqrt{3}}_{\mathcal{A}_{ABCD}} + \underbrace{\cancel{2}\cdot\frac{1}{\cancel{2}}\,x\frac{\sqrt{3}}{2}\cdot\frac{x}{2}}_{\mathcal{A}_{EHC}} = \underbrace{8\sqrt{3}}_{\mathcal{A}_{ABCDE}} \Rightarrow \cancel{\sqrt{3}}\,x + \frac{\cancel{\sqrt{3}}}{4}x^2 - 8\cancel{\sqrt{3}} = 0$$

$$x^2 + 4x - 32 = 0$$

$$(x+8)(x-4) = 0 \quad \begin{cases} x = -8 & \text{N.A. perché} < 0 \\ x = 4 & \overline{CE} \end{cases}$$

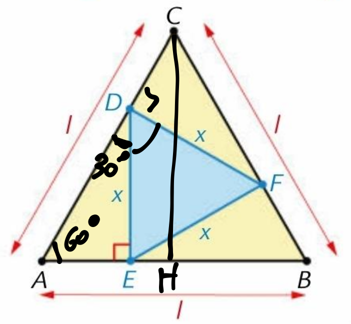
$$2P_{ABCDE} = 1 + 4 + 1 + 4 + 2\overline{HC} = 10 + \cancel{2}\cdot 4\cdot\frac{\sqrt{3}}{\cancel{2}} = (10 + 4\sqrt{3})\ \text{cm}$$

146 Sia ABC un triangolo equilatero il cui lato misura l e sia DEF un altro triangolo equilatero in esso inscritto, con il lato ED perpendicolare ad AB.
Determina:

a. la misura del lato del triangolo DEF;

b. il rapporto fra le aree di ABC e DEF.

$\left[\frac{l\sqrt{3}}{3}; 3\right]$



$$0 < x < \underbrace{l\frac{\sqrt{3}}{2}}_{\overline{CH}}$$

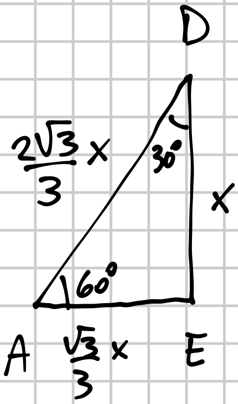
$\overline{AE} =$

$$\overline{DE} = \overline{AE} \cdot \sqrt{3}$$

$$\Downarrow$$

$$\overline{AE} = \frac{\overline{DE}}{\sqrt{3}} = \frac{x}{\sqrt{3}} = \frac{\sqrt{3}}{3}x$$

$$\overline{AD} = 2\frac{x}{\sqrt{3}} = \frac{2\sqrt{3}}{3}x$$



$$\overline{DE} = \overline{AD} \cdot \frac{\sqrt{3}}{2}$$

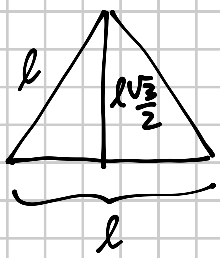
$$\Downarrow$$

$$\overline{AD} = \frac{2}{\sqrt{3}}\overline{DE} = \frac{2\sqrt{3}}{3}\overline{DE} = \frac{2\sqrt{3}}{3}x$$

$$l = \overline{AD} + \underset{\overline{AE}}{\overset{}{\overline{DC}}} \Rightarrow \quad l = \frac{2\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}x$$

$$\frac{\cancel{3}\sqrt{3}}{\cancel{3}}x = l \qquad x = \frac{l}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{l\frac{\sqrt{3}}{3}}$$

Dato un triangolo equilatero di lato l:



$$\mathcal{A} = \frac{1}{2}l \cdot l\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}l^2$$

$\mathcal{A}$ è dirett. proporzionale a l^2

TRIANGOLO 1 : $\mathcal{A}_1 = \frac{\sqrt{3}}{4}l_1^2$

TRIANGOLO 2 : $\mathcal{A}_2 = \frac{\sqrt{3}}{4}l_2^2$

$$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \frac{l_1^2}{l_2^2}$$

$$\frac{\mathcal{A}_{ABC}}{\mathcal{A}_{DEF}} = \frac{l^2}{x^2} = \frac{\cancel{l^2}}{\cancel{l^2} \cdot \frac{3}{9}} = \frac{9}{3} = 3$$