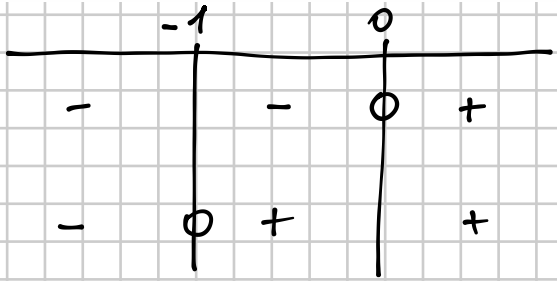
19/4/2021

304 $|x| + |x+1| = 2 - x^2$ $[-1+\sqrt{2}; -1]$



$x > 0$

$x + 1 > 0 \Rightarrow x > -1$

$$\begin{cases} x \leq -1 \\ -x - (x+1) = 2 - x^2 \end{cases} \vee \begin{cases} -1 \leq x \leq 0 \\ -x + x + 1 = 2 - x^2 \end{cases} \vee \begin{cases} x \geq 0 \\ x + x + 1 = 2 - x^2 \end{cases}$$

$$\begin{cases} x \leq -1 \\ -2x - 1 = 2 - x^2 \end{cases} \vee \begin{cases} -1 \leq x \leq 0 \\ x^2 = 1 \end{cases} \vee \begin{cases} x \geq 0 \\ x^2 + 2x - 1 = 0 \end{cases}$$

$x^2 - 2x - 3 = 0$

$\frac{\Delta}{4} = 1 + 3 = 4$

$x = 1 \pm 2 = \begin{cases} -1 \\ 3 \text{ N.A.} \end{cases}$

$x = \pm 1 \begin{cases} -1 \\ 1 \text{ N.A.} \end{cases}$

$\frac{\Delta}{4} = 1 + 1 = 2$ N.A.

$x = -1 \pm \sqrt{2} = \begin{cases} -1-\sqrt{2} \\ -1+\sqrt{2} \end{cases}$

$x = -1 \vee x = -1 \vee x = -1 + \sqrt{2}$

$$\boxed{x = -1 \quad \vee \quad x = \sqrt{2} - 1}$$

319 $\begin{cases} |x-2| = 2|y| \\ x - 3y - 3 = 0 \end{cases}$

$|f(x)| = |g(x)|$

$\Downarrow$

$f(x) = \pm g(x)$

$f(x) = g(x) \ \vee \ f(x) = -g(x)$

$\begin{cases} |x-2| = 2|y| \\ 3y = x-3 \end{cases}$ $\quad$ $\begin{cases} |x-2| = 2\left|\frac{x-3}{3}\right| \\ y = \frac{x-3}{3} \end{cases}$

$\begin{cases} |x-2| = \frac{2}{3}|x-3| \\ y = \frac{x-3}{3} \end{cases} \Rightarrow \begin{cases} x-2 = \pm\frac{2}{3}(x-3) \\ y = \frac{x-3}{3} \end{cases}$

$\Downarrow$

① $\begin{cases} x-2 = -\frac{2}{3}(x-3) \\ y = \frac{x-3}{3} \end{cases}$ $\vee$ ② $\begin{cases} x-2 = \frac{2}{3}(x-3) \\ y = \frac{x-3}{3} \end{cases}$

① $3x - 6 = -2x + 6$

$5x = 12$

$\begin{cases} x = \frac{12}{5} \\ y = \frac{\frac{12}{5} - 3}{3} = \frac{-\frac{3}{5}}{3} = -\frac{1}{5} \end{cases}$

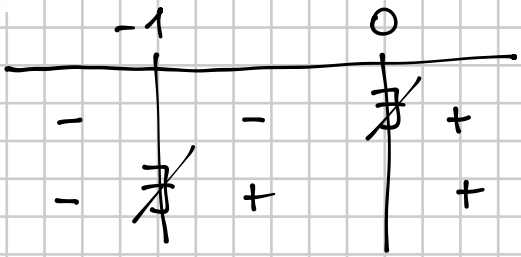
② $3x - 6 = 2x - 6$

$\begin{cases} x = 0 \\ y = \frac{0-3}{3} = -1 \end{cases}$

$\boxed{\begin{cases} x = \frac{12}{5} \\ y = -\frac{1}{5} \end{cases} \vee \begin{cases} x = 0 \\ y = -1 \end{cases}}$

289 $\dfrac{1}{|x|}+\dfrac{2}{|x+1|}=2$

C.E. $x\neq 0$
$x\neq -1$



$x>0$

$x+1>0 \Rightarrow x>-1$

$$\begin{cases} x<-1 \\ -\dfrac{1}{x}-\dfrac{2}{x+1}=2 \end{cases} \vee \begin{cases} -1<x<0 \\ -\dfrac{1}{x}+\dfrac{2}{x+1}=2 \end{cases} \vee \begin{cases} x>0 \\ \dfrac{1}{x}+\dfrac{2}{x+1}=2 \end{cases}$$

$$\begin{cases} x<-1 \\ \dfrac{-x-1-2x}{x(x+1)}=\dfrac{2x(x+1)}{x(x+1)} \end{cases} \vee \begin{cases} -1<x<0 \\ \dfrac{-x-1+2x}{x(x+1)}=\dfrac{2x(x+1)}{x(x+1)} \end{cases} \vee \begin{cases} x>0 \\ \dfrac{x+1+2x}{x(x+1)}=\dfrac{2x(x+1)}{x(x+1)} \end{cases}$$

$$\begin{cases} x<-1 \\ 2x^2+5x+1=0 \end{cases} \vee \begin{cases} -1<x<0 \\ 2x^2+x+1=0 \end{cases} \vee \begin{cases} x>0 \\ 2x^2-x-1=0 \end{cases}$$

$\Delta = 25-8=17$

$\Delta = 1-8<0$

$\Delta = 1+8=9$

$x=\dfrac{-5\pm\sqrt{17}}{4}$

$=\begin{cases} \dfrac{-5-\sqrt{17}}{4} \\ \dfrac{-5+\sqrt{17}}{4} \text{ N.A.} \end{cases}$

$\varnothing$ IMP.

$x=\dfrac{1\pm 3}{4}=$

$=\begin{cases} -\frac{1}{2} \text{ N.A.} \\ 1 \end{cases}$

$$\boxed{x=\frac{-5-\sqrt{17}}{4} \quad \vee \quad x=1}$$

317 $$\frac{1}{x^2-4x+4}+\frac{1}{|x-2|}=\frac{2}{4-x}$$

C.E.
$x \neq 2$
$x \neq 4$

$(x-2)^2$

$x-2>0 \Rightarrow x>2$

① $\begin{cases} x<2 \\ \frac{1}{(x-2)^2}-\frac{1}{x-2}=\frac{2}{4-x} \end{cases}$

∨

② $\begin{cases} x>2 \\ \frac{1}{(x-2)^2}+\frac{1}{x-2}=\frac{2}{4-x} \end{cases}$

① $\frac{4-x-(x-2)(4-x)}{(x-2)^2(4-x)}=\frac{2(x-2)^2}{(x-2)^2(4-x)}$

$4-x-(4x-x^2-8+2x)=2(x^2-4x+4)$

$4-x-4x+x^2+8-2x=2x^2-8x+8$

$x^2-x-4=0$

$\Delta=1+16=17$

$x=\frac{1\pm\sqrt{17}}{2}=\begin{cases}\frac{1+\sqrt{17}}{2} & \text{N.A.}\\ \frac{1-\sqrt{17}}{2}\end{cases}$

$x=\frac{1-\sqrt{17}}{2}$

$$\begin{cases} x > 2 \\ \dfrac{1}{(x-2)^2} + \dfrac{1}{x-2} = \dfrac{2}{4-x} \end{cases}$$

$$\frac{4-x+(x-2)(4-x)}{(x-2)^2(4-x)} = \frac{2(x-2)^2}{(x-2)^2(4-x)}$$

$$4-x+4x-x^2-8+2x = 2x^2+8-8x$$

$$3x^2-13x+12=0$$

$$\Delta = 169-144 = 25$$

$$x = \frac{13 \pm 5}{6} = \begin{cases} \frac{8}{6} = \frac{4}{3} \text{ N.A.} \\ \frac{18}{6} = 3 \end{cases}$$

$$\boxed{x = \frac{1-\sqrt{17}}{2} \vee x = 3}$$