25/9/2020

89 Considera la funzione

$$y = f(x) = 2\left|\log_2 x\right| + \log_2 2x - 2.$$

a. Trova il dominio, studia il segno di $f(x)$ e disegna il grafico di $f(x)$.

b. La funzione è monotòna?
È invertibile?
Se non lo è in tutto il suo dominio, effettua una restrizione, trova $f^{-1}(x)$ e mostra che $f(f^{-1}(x)) = x$.

c. Disegna i grafici di $y = f(x) + 1$ e di $y = f(x + 1)$.

[a) D: $x > 0$; $f(x) > 0$ per $0 < x < \frac{1}{2} \vee x > \sqrt[3]{2}$; b) per $x \geq 1$, $f^{-1}(x) = 2^{\frac{x+1}{3}}$]

a)

$$f(x) = 2|\log_2 x| + \log_2 2x - 2 =$$

$D =]0, +\infty[$

$$= 2|\log_2 x| + \underbrace{\log_2 2}_{1} + \log_2 x - 2 =$$

$$= 2|\log_2 x| + \log_2 x - 1$$

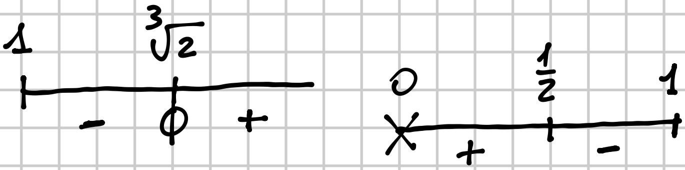
$$= \begin{cases} 2\log_2 x + \log_2 x - 1 & \text{se } \log_2 x \geq 0 \\ -2\log_2 x + \log_2 x - 1 & \text{se } \log_2 x < 0 \end{cases}$$

$$f(x) = \begin{cases} 3\log_2 x - 1 & \text{se } x \geq 1 \\ -\log_2 x - 1 & \text{se } 0 < x < 1 \end{cases}$$

SEGNO

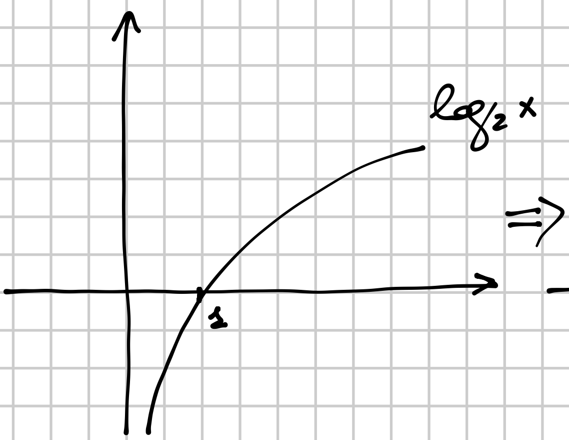
$f(x) > 0$ ① $\begin{cases} 3\log_2 x - 1 > 0 \\ x \geq 1 \end{cases}$ $\vee$ ② $\begin{cases} -\log_2 x - 1 > 0 \\ 0 < x < 1 \end{cases}$

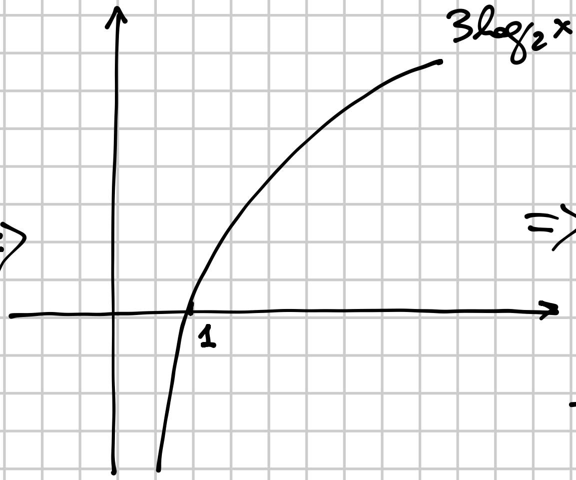
① $\log_2 x > \frac{1}{3} \Rightarrow x > 2^{\frac{1}{3}} = \sqrt[3]{2}$

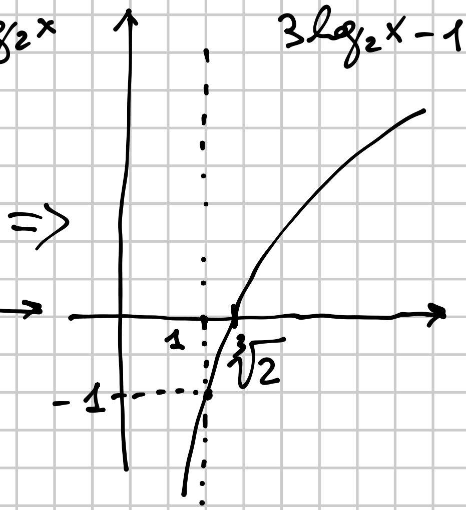


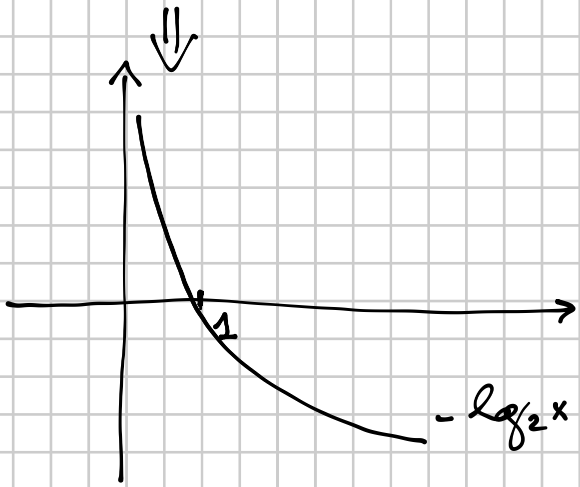
② $-\log_2 x - 1 > 0 \Rightarrow -\log_2 x > 1$ $\log_2 x < -1 \Rightarrow 0 < x < 2^{-1} = \frac{1}{2}$

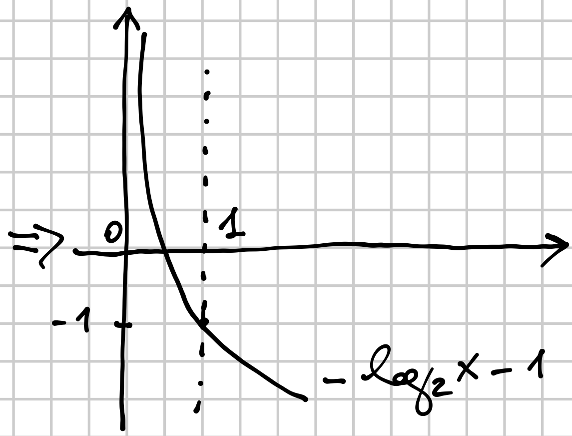
$$f(x) = \begin{cases} 3\log_2 x - 1 & \text{se} \quad x \geq 1 \\ -\log_2 x - 1 & \text{se} \quad 0 < x < 1 \end{cases}$$

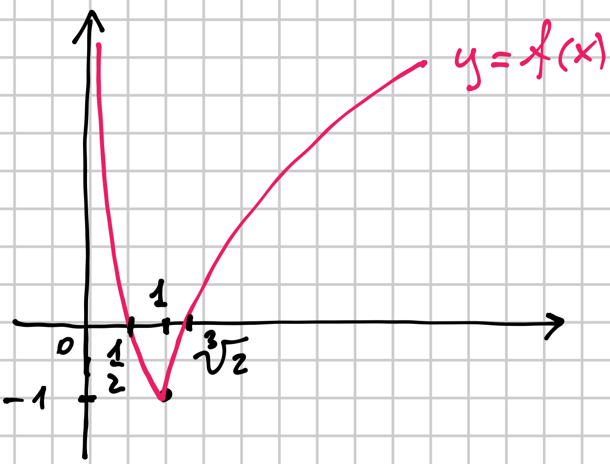












$f(x) > 0$ per $x \in \left]0, \frac{1}{2}\right[\cup \left]\sqrt[3]{2}, +\infty\right[$

$f(x) < 0$ per $x \in \left]\frac{1}{2}, \sqrt[3]{2}\right[$

$f(x) = 0$ per $x = \frac{1}{2}$ o $x = \sqrt[3]{2}$ (gli zeri sono $\frac{1}{2}$ e $\sqrt[3]{2}$)

f NON è monotona, quindi non invertibile (per essere invertibile deve essere strettamente monotona)

Tuttavia è

strett. decrescente in $]0,1[$ (o in $]0,1]$)

strett. crescente in $]1,+\infty[$ (o in $[1,+\infty[$)

È invertibile la restrizione di f a ciascuno dei due intervalli

$$f_{|[1,+\infty[}(x) = 3\log_2 x - 1$$

$$y = 3\log_2 x - 1$$

$$y+1 = 3\log_2 x \qquad \log_2 x = \frac{y+1}{3}$$

$$x = 2^{\frac{y+1}{3}}$$

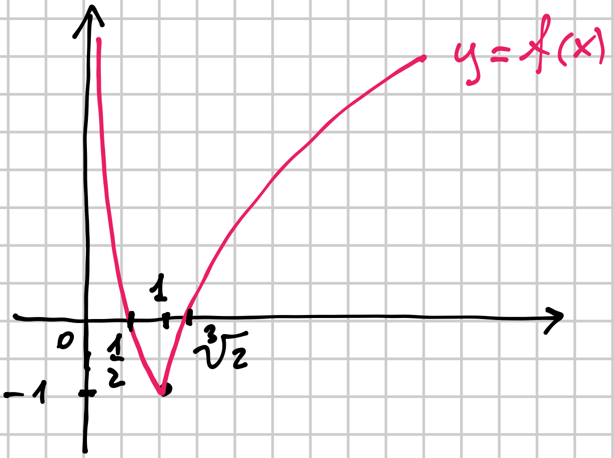
$$\Downarrow$$

$$y = 2^{\frac{x+1}{3}}$$

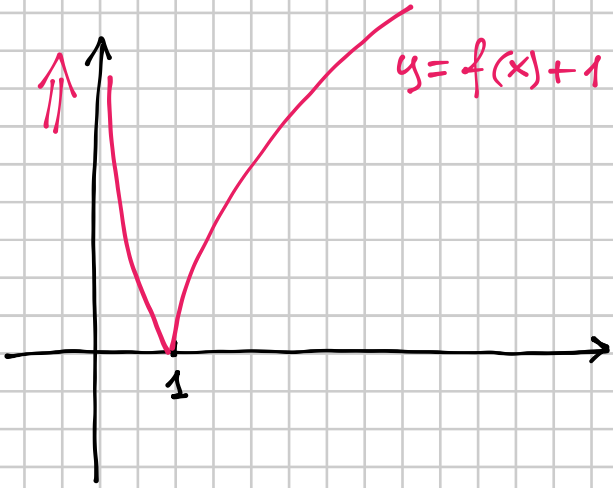
$$f^{-1} : [-1,+\infty[\to \mathbb{R} \qquad f^{-1}(x) = 2^{\frac{x+1}{3}}$$

$$f(f^{-1}(x)) = f\left(2^{\frac{x+1}{3}}\right) = 3\log_2\left(2^{\frac{x+1}{3}}\right) - 1 =$$

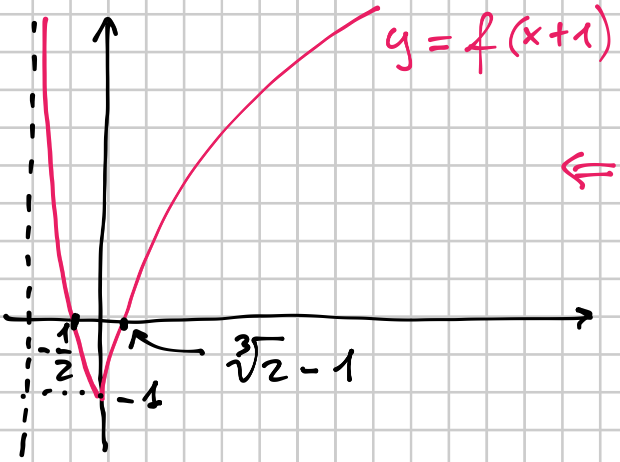
$$= \cancel{3} \cdot \frac{x+1}{\cancel{3}} - 1 = x + \cancel{1} - \cancel{1} = x$$



y = f(x)
1
0
1/2
∛2
-1



⇑
y = f(x) + 1
1



y = f(x+1)
⇐
-1/2
∛2 - 1
-1