6/10/2020

5. $-n^3 + 2n^2 - 4$

$$\lim_{n \to +\infty} \left(-n^3 + 2n^2 - 4\right) = -\infty + \infty \qquad F.I.$$

$$\lim_{n \to +\infty} \left(-n^3 + 2n^2 - 4\right) = \lim_{n \to +\infty} \underbrace{n^3}_{+\infty} \underbrace{\left(-1 + \overset{\to 0}{\frac{2}{n}} - \overset{\to 0}{\frac{4}{n^3}}\right)}_{-1} =$$

$$= +\infty \cdot (-1) = -\infty$$

6. $3 - \dfrac{2}{5n+1}$

$$\lim_{n \to +\infty} \left(3 - \underbrace{\frac{2}{\underbrace{5n+1}_{+\infty}}}_{0}\right) = 3$$

10. $\dfrac{n^2-5n+1}{4n^2-3n+5}$

11. $\dfrac{3n-7}{8n^2+4n+5}$

12. $\dfrac{2n^3-5n+1}{3n^2+n-2}$

13. $\dfrac{4n^2+1}{3n^2+n-4}$

10) $$\lim_{n\to+\infty}\frac{n^2-5n+1}{4n^2-3n+5}=\frac{+\infty-\infty+1}{+\infty-\infty+5}\quad \text{F.I.}$$

$$=\frac{+\infty}{+\infty}$$

$$\lim_{n\to+\infty}\frac{\cancel{n^2}\left(1-\frac{5}{n}+\frac{1}{n^2}\right)}{\cancel{n^2}\left(4-\frac{3}{n}+\frac{5}{n^2}\right)}=\frac{1}{4}$$

11) $$\lim_{n\to+\infty}\frac{3n-7}{8n^2+4n+5}=\frac{+\infty}{+\infty}\quad \text{F.I.}$$

$$\lim_{n\to+\infty}\frac{\cancel{n}\left(3-\frac{7}{n}\right)}{n^{\cancel{2}}\left(8+\frac{4}{n}+\frac{5}{n^2}\right)}=\frac{3}{+\infty}=0$$

12) $$\lim_{n\to+\infty}\frac{2n^3-5n+1}{3n^2+n-2}=\frac{\infty}{\infty}\quad \text{F.I.}$$

$$=\lim_{n\to+\infty}\frac{n^{\cancel{3}}\left(2-\frac{5}{n^2}+\frac{1}{n^3}\right)}{\cancel{n^2}\left(3+\frac{1}{n}-\frac{2}{n^2}\right)}=\frac{+\infty\cdot 2}{3}=+\infty$$

13) $$\lim_{n\to+\infty}\frac{4n^2+1}{3n^2+n-4}=\frac{+\infty}{+\infty}\quad \text{F.I.}$$

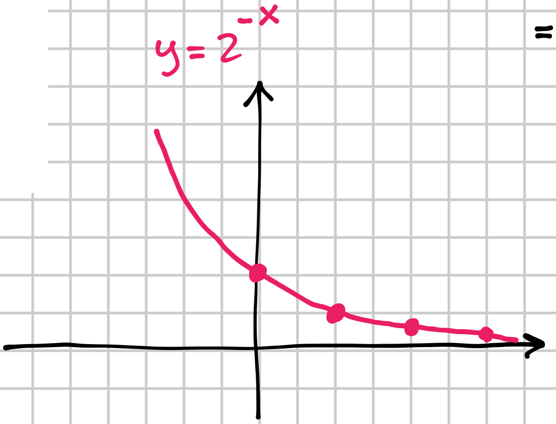
$$=\lim_{n\to+\infty}\frac{\cancel{n^2}\left(4+\frac{1}{n^2}\right)}{\cancel{n^2}\left(3+\frac{1}{n}-\frac{4}{n^2}\right)}=\frac{4}{3}$$

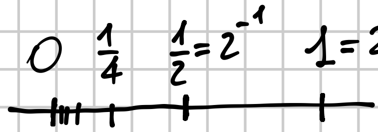
8. 2^{-n}

9. $\dfrac{1}{\sqrt{1+n^2}}$

8) $\lim\limits_{n\to+\infty} 2^{-n} = \lim\limits_{n\to+\infty} \dfrac{1}{2^n} =$

$= \dfrac{1}{+\infty} = 0$





9) $\lim\limits_{n\to+\infty} \dfrac{1}{\sqrt{1+n^2}} = \dfrac{1}{\sqrt{+\infty}} = \dfrac{1}{+\infty} = 0$

ESEMPIO → TEOREMA DELLA PERMANENZA DEL SEGNO

$a_n = \dfrac{1}{n} - \dfrac{1}{10}$ $\qquad \lim\limits_{n\to+\infty} \left(\dfrac{1}{n} - \dfrac{1}{10}\right) = -\dfrac{1}{10} < 0$

$a_1 = 1 - \dfrac{1}{10} = \dfrac{9}{10} > 0$

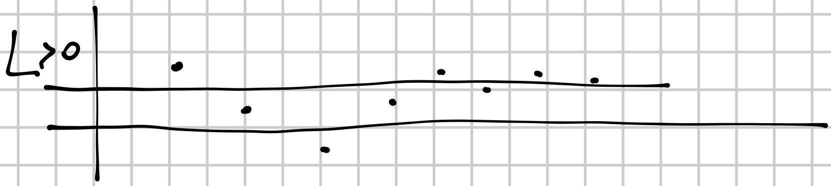
$a_2 = 2 - \dfrac{1}{10} = \dfrac{19}{10} > 0$

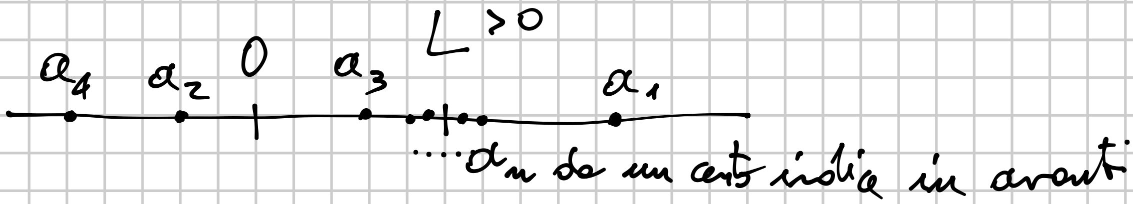
$\vdots$

siccome il limite è < 0, il teorema dice che da un certo punto in avanti tutti i termini della successione sono negativi:

prendo $m = 11 \quad \forall n \geq m \quad a_n < 0$

$\dfrac{1}{11} - \dfrac{1}{10} < 0$ e così per tutti i termini che seguono





7. $\dfrac{(-1)^n}{n+1}$

$$\lim_{n\to+\infty} \frac{(-1)^n}{n+1} = ?$$

$$(-1)^n = \begin{cases} 1 & \text{se } n \text{ pari} \\ -1 & \text{se } n \text{ dispari} \end{cases}$$

$\lim_{n\to+\infty} (-1)^n$ <u>NON ESISTE</u> !

$$-1 \leq (-1)^n \leq 1 \qquad \forall n$$

$$\underbrace{\frac{-1}{n+1}}_{\downarrow \atop 0} \leq \underbrace{\frac{(-1)^n}{n+1}}_{\downarrow \atop 0} \leq \underbrace{\frac{1}{n+1}}_{\downarrow \atop 0} \qquad n\to+\infty$$

0 per il TH. DEI DUE CARABINIERI

$$\lim_{n\to+\infty} \frac{(-1)^n}{n+1} = 0$$

ALTRO ESEMPIO

$$\lim_{n\to+\infty} \frac{\sin\frac{1}{n}}{n} = 0$$

$$-1 \leq \sin\frac{1}{n} \leq 1$$

$$\underset{\downarrow\, 0}{-\frac{1}{n}} \leq \underbrace{\frac{\sin\frac{1}{n}}{n}}_{\downarrow\, 0} \leq \underset{\downarrow\, 0}{\frac{1}{n}}$$

per il TH. DEI CARABINIERI

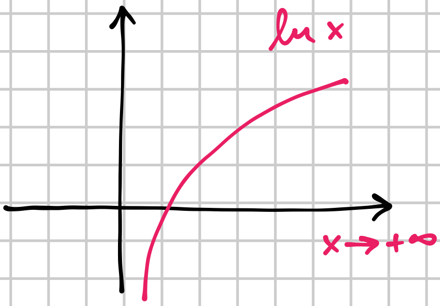
21. $\ln(n^2+n)$

$$\lim_{n\to+\infty} \ln(n^2+n) =$$

$$= \ln(+\infty) = +\infty$$

significa

$$\lim_{n\to+\infty} \ln(n) = +\infty$$



23. $\sqrt{n+1} - \sqrt{n-2}$

$$\lim_{n\to+\infty} \left(\sqrt{n+1} - \sqrt{n-2}\right) = +\infty - \infty \quad \text{F.I.}$$

$$\lim_{n\to+\infty} \left[\left(\sqrt{n+1} - \sqrt{n-2}\right) \cdot \frac{\sqrt{n+1}+\sqrt{n-2}}{\sqrt{n+1}+\sqrt{n-2}}\right] = \lim_{n\to+\infty} \frac{n+1-(n-2)}{\sqrt{n+1}+\sqrt{n-2}} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{n}+1-\cancel{n}+2}{\sqrt{n+1}+\sqrt{n-2}} = \frac{3}{+\infty+\infty} = \frac{3}{+\infty} = 0$$