27/10/2020

113 $\lim_{x\to+\infty} \frac{1+\cos x}{x^2}$ [0]

SI USA IL
TH. DEI CARABINIERI

$$-1 \leq \cos x \leq 1 \qquad \forall x \in \mathbb{R}$$

$$\Downarrow$$

$$0 \leq 1+\cos x \leq 2 \qquad \forall x \in \mathbb{R}$$

$$\frac{0}{x^2} \leq \frac{1+\cos x}{x^2} \leq \frac{2}{x^2} \qquad \forall x \in \mathbb{R}\setminus\{0\}$$

$$\underset{\downarrow \atop 0}{0} \leq \underset{\downarrow \atop 0}{\frac{1+\cos x}{x^2}} \leq \underset{\downarrow \atop 0}{\underbrace{\frac{2}{x^2}}}$$

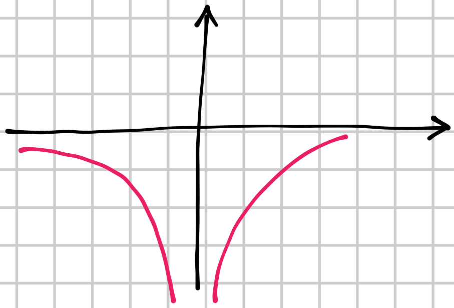
per $x\to+\infty$

per il th. dei carabinieri

$$\lim_{x\to+\infty} \frac{1+\cos x}{x^2} = 0$$

28 $\lim_{x \to -\infty} \frac{-2}{x^4}$ [0]

$$\lim_{x \to -\infty} \frac{-2}{x^4} = \frac{-2}{(-\infty)^4} = \frac{-2}{+\infty} = 0^-$$



214 $\lim_{x \to -2} \frac{2x^2+3x-2}{x^2+2x} = \frac{0}{0}$ F. I. $\left[\frac{5}{2}\right]$

$$= \lim_{x \to -2} \frac{(2x-1)\cancel{(x+2)}}{x\cancel{(x+2)}} = \lim_{x \to -2} \frac{2x-1}{x} = \frac{-5}{-2} = \frac{5}{2}$$

	2	3	−2
−2		−4	2
	2	−1	//

206 $\lim_{x \to -\infty} \frac{\sqrt{9x^4+5x^2}}{(x+2)^2}$ [3]

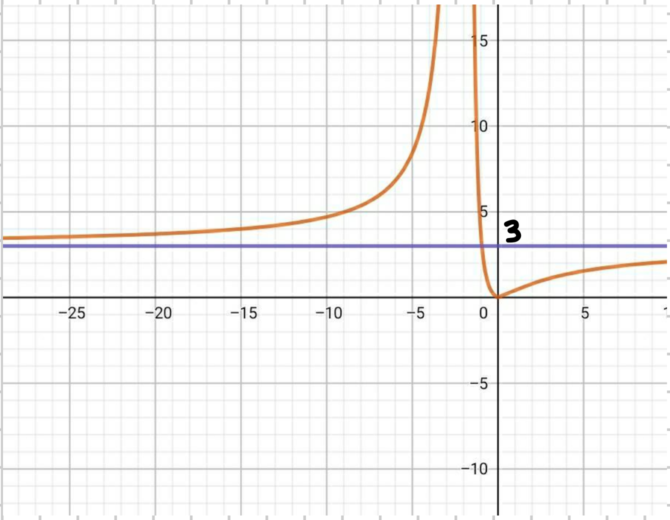
207 $\lim_{x \to -\infty} \frac{\sqrt{x^2-1}+2x}{8x-4}$ $\left[\frac{1}{8}\right]$

206) $\lim_{x \to -\infty} \frac{\sqrt{9x^4+5x^2}}{(x+2)^2} = \frac{+\infty}{+\infty}$ F.I.

$$\lim_{x \to -\infty} \frac{\sqrt{x^4\left(9+\frac{5}{x^2}\right)}}{\left[x\left(1+\frac{2}{x}\right)\right]^2} = \lim_{x \to -\infty} \frac{\cancel{x^2}\sqrt{9+\frac{5}{x^2}}}{\cancel{x^2}\left(1+\frac{2}{x}\right)^2} = \frac{\sqrt{9}}{1^2} = 3$$

$|x^2|$

$\frac{5}{x^2} \to 0$, $\frac{2}{x} \to 0$



207) $\lim_{x \to -\infty} \frac{\sqrt{x^2-1}+2x}{8x-4} = \frac{+\infty-\infty}{+\infty}$ F.I.

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}+2x}{8x-4} = \lim_{x \to -\infty} \frac{\overbrace{-x}^{|x|}\sqrt{1-\frac{1}{x^2}}+2x}{x\left(8-\frac{4}{x}\right)} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{x}\left[-\sqrt{1-\frac{1}{x^2}}+2\right]}{\cancel{x}\left(8-\frac{4}{x}\right)} = \frac{-1+2}{8} = \frac{1}{8}$$

$\frac{1}{x^2} \to 0$, $\frac{4}{x} \to 0$

ALTERNATIVA

$$\lim_{x\to-\infty} \frac{\sqrt{x^2-1}+2x}{8x-4}\cdot\frac{\sqrt{x^2-1}-2x}{\sqrt{x^2-1}-2x} =$$

$$= \lim_{x\to-\infty} \frac{x^2-1-4x^2}{(8x-4)\left[\sqrt{x^2-1}-2x\right]} = \lim_{x\to-\infty} \frac{-3x^2-1}{(8x-4)\left[\sqrt{x^2\left(1-\frac{1}{x^2}\right)}-2x\right]} =$$

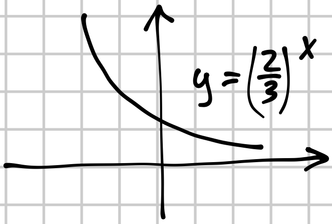
$$= \lim_{x\to-\infty} \frac{-x^2\left(3+\frac{1}{x^2}\right)}{x\left(8-\frac{4}{x}\right)\left[-x\left(\sqrt{1-\frac{1}{x^2}}+2\right)\right]} =$$

$$= \lim_{x\to-\infty} \frac{\cancel{-x^2}\left(3+\frac{1}{x^2}\right)}{\cancel{-x^2}\left(8-\frac{4}{x}\right)\left(\sqrt{1-\frac{1}{x^2}}+2\right)} = \frac{\cancel{3}}{8\cdot\cancel{(1+2)}} = \frac{1}{8}$$

$\frac{1}{x^2} \to 0$, $\frac{4}{x} \to 0$, $\frac{1}{x^2} \to 0$

231 $\lim_{x\to 1^+}\left(\frac{3x^2-4x+1}{x^2+x-2}\right)^{\frac{1}{1-x}} = \left(\frac{0}{0}\right)^{\infty}$ F.I. [$+\infty$]

$\frac{1}{1-x} \longrightarrow -\infty$ per $x\to 1^+$



A PARTE

$$\lim_{x\to 1^+}\frac{3x^2-4x+1}{x^2+x-2} = \lim_{x\to 1^+}\frac{\cancel{(x-1)}(3x-1)}{\cancel{(x-1)}(x+2)} = \frac{3-1}{1+2} = \frac{2}{3}$$

$$\begin{array}{c|cc|c} & 3 & -4 & 1 \\ 1 & & 3 & -1 \\ \hline & 3 & -1 & // \end{array}$$

$$\lim_{x\to 1^+}\left(\frac{3x^2-4x+1}{x^2+x-2}\right)^{\frac{1}{1-x}} = \left(\frac{2}{3}\right)^{-\infty} = +\infty$$

243 $\lim_{x\to 2}\frac{\sqrt{x^2-2x+9}-3}{x-2} = \frac{0}{0}$ F.I. $\left[\frac{1}{3}\right]$

$$\lim_{x\to 2}\frac{\sqrt{x^2-2x+9}-3}{x-2}\cdot\frac{\sqrt{x^2-2x+9}+3}{\sqrt{x^2-2x+9}+3} =$$

$$= \lim_{x\to 2}\frac{x^2-2x+\cancel{9}-\cancel{9}}{(x-2)\left[\sqrt{x^2-2x+9}+3\right]} = \lim_{x\to 2}\frac{x\cancel{(x-2)}}{\cancel{(x-2)}\left[\sqrt{x^2-2x+9}+3\right]} =$$

$$= \frac{2}{\sqrt{9}+3} = \frac{2}{6} = \frac{1}{3}$$

FORME INDETERMINATE

$0 \cdot \infty \quad \infty \cdot 0 \qquad \frac{\infty}{\infty} \qquad \frac{0}{0} \qquad +\infty - \infty \quad -\infty + \infty$

$$0 \cdot \infty = \frac{1}{\frac{1}{0}} \cdot \infty = \frac{\infty}{\infty}$$

FORME INDETERMINATE ESPONENZIALI

$$\underbrace{0^0 \quad 1^\infty \quad \infty^0}$$

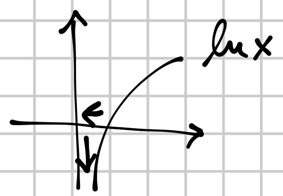
RICONDUCIBILI A QUELLE CHE GIÀ CONOSCIAMO

248 $\lim_{x \to 0^+} (2x)^{\frac{2}{\ln 2x}}$ $[e^2]$

PREMESSA IMPORTANTE

$$\boxed{\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e}$$

$$\lim_{x \to 0^+} (2x)^{\frac{2}{\ln 2x}} = 0^0$$



TRUCCHETTO $[f(x)]^{g(x)} = e^{g(x) \cdot \ln f(x)}$

$$\lim_{x \to 0^+} (2x)^{\frac{2}{\ln 2x}} = \lim_{x \to 0^+} e^{\frac{2}{\cancel{\ln 2x}} \cdot \cancel{\ln 2x}} = e^2$$

253 $\lim_{x\to 0^+} x^{-\frac{1}{\ln x^2}} = 0^0$ $\left[\frac{1}{\sqrt{e}}\right]$

$$= \lim_{x\to 0^+} e^{\ln x^{-\frac{1}{\ln x^2}}} = \lim_{x\to 0^+} e^{-\frac{1}{\ln x^2}\cdot \ln x} =$$

$$= \lim_{x\to 0^+} e^{-\frac{1}{2\cancel{\ln x}}\cdot \cancel{\ln x}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

256 $\lim_{x\to +\infty} x^{\frac{1}{1+\ln x}} = \infty^0$ F.I. $[e]$

$$= \lim_{x\to +\infty} e^{\ln x^{\frac{1}{1+\ln x}}} = \lim_{x\to +\infty} e^{\overbrace{\frac{1}{1+\ln x}\cdot \ln x}^{0\cdot\infty}} = e^1 = e$$

A PARTE

$$\lim_{x\to +\infty} \frac{\ln x}{1+\ln x} = \lim_{x\to +\infty} \frac{1+\ln x - 1}{1+\ln x} = \lim_{x\to +\infty} \left(\frac{1+\ln x}{1+\ln x} - \frac{1}{1+\ln x}\right) =$$

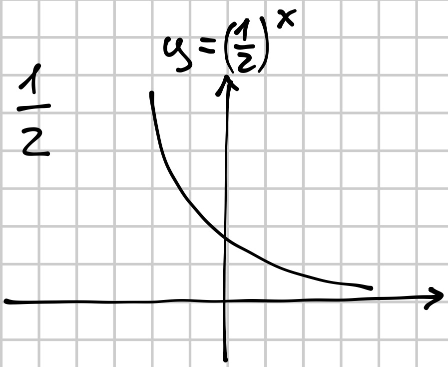
$$= \lim_{x\to +\infty} \left(1 - \underbrace{\frac{1}{\underbrace{1+\ln x}_{+\infty}}}_{\to 0}\right) = 1$$

un'altra possibilità

$$\lim_{x\to +\infty} \frac{\ln x}{1+\ln x} = \lim_{x\to +\infty} \frac{1}{\frac{1+\ln x}{\ln x}} = \lim_{x\to +\infty} \frac{1}{\underbrace{\frac{1}{\ln x}}_{0} + 1} = 1$$

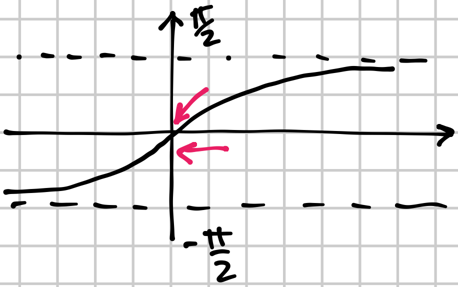
307 $\lim\limits_{x\to+\infty}\left(\dfrac{x+1}{2x-3}\right)^{x-1} = \left(\dfrac{1}{2}\right)^{+\infty} = 0^+$ [0^+]

$$\lim_{x\to+\infty}\frac{x+1}{2x-3}=\frac{1}{2}$$

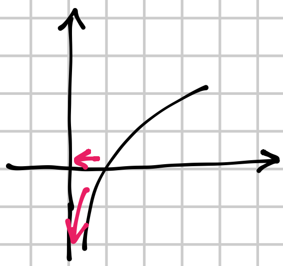


308 $\lim\limits_{x\to+\infty}\left(\dfrac{4x^2-x}{x+1}\right)^{x^2} = (+\infty)^{+\infty} = +\infty$ [$+\infty$]

313 $\lim\limits_{x\to-\infty}\ln\arctan\dfrac{x-2}{1-x^2} =$ [$-\infty$]



$= \ln\arctan 0^+ = \ln 0^+ = -\infty$



$$x\to-\infty \qquad \frac{x-2}{1-x^2}\to 0^+$$

324 $\lim_{x\to\pm\infty} \frac{2x-\sqrt{x^2-1}}{4x-1}$

$\left[\text{se } x\to+\infty: \frac{1}{4}; \text{ se } x\to-\infty: \frac{3}{4}\right]$

$$\lim_{x\to+\infty} \frac{2x-\sqrt{x^2-1}}{4x-1} = \frac{+\infty-\infty}{+\infty}$$

$$= \lim_{x\to+\infty} \frac{2x-\sqrt{x^2\left(1-\frac{1}{x^2}\right)}}{4x-1} = \lim_{x\to+\infty} \frac{2x-\overset{|x|}{x}\sqrt{1-\frac{1}{x^2}}}{x\left(4-\frac{1}{x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\left(2-\sqrt{1-\underset{\to 0}{\frac{1}{x^2}}}\right)}{\cancel{x}\left(4-\underset{\to 0}{\frac{1}{x}}\right)} = \frac{2-1}{4} = \frac{1}{4}$$

$$\lim_{x\to-\infty} \frac{2x-\sqrt{x^2-1}}{4x-1} = \frac{-\infty-\infty}{-\infty} = \frac{-\infty}{-\infty} \quad \text{F.I.} \qquad |x| = -x \text{ per } x\to-\infty$$

$$= \lim_{x\to-\infty} \frac{2x-\sqrt{x^2\left(1-\frac{1}{x^2}\right)}}{x\left(4-\frac{1}{x}\right)} = \lim_{x\to-\infty} \frac{2x-|x|\sqrt{1-\frac{1}{x^2}}}{x\left(4-\frac{1}{x}\right)} =$$

$$= \lim_{x\to-\infty} \frac{2x+x\sqrt{1-\frac{1}{x^2}}}{x\left(4-\frac{1}{x}\right)} = \lim_{x\to-\infty} \frac{\cancel{x}\left(2+\sqrt{1-\underset{\to 0}{\frac{1}{x^2}}}\right)}{\cancel{x}\left(4-\underset{\to 0}{\frac{1}{x}}\right)} = \frac{3}{4}$$