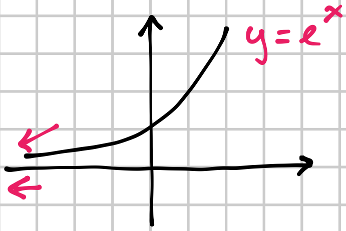
29/10/2020

281 $\displaystyle \lim_{x\to-\infty}\left(\frac{x^3+1}{x+2}\right)^{x^3+4x^2}$ $[0^+]$

$$\lim_{x\to-\infty}\left(\frac{x^3+1}{x+2}\right)^{x^3+4x^2} =$$

$$= \lim_{x\to-\infty} e^{\underbrace{(x^3+4x^2)}_{-\infty}\cdot \underbrace{\ln\left(\overset{+\infty}{\frac{x^3+1}{x+2}}\right)}_{+\infty}} = e^{-\infty} = 0^+$$



$$\frac{x^3+1}{x+2} = \frac{x^{\cancel{3}\,2}\left(1+\frac{1}{x^3}\right)}{\cancel{x}\left(1+\frac{1}{x^2}\right)} \longrightarrow +\infty$$
per $x\to-\infty$

$$[f(x)]^{g(x)} = e^{g(x)\cdot \ln f(x)}$$

279 $\displaystyle \lim_{x\to-\infty}\frac{8x+2}{x-\sqrt{x^2-3}} = \frac{-\infty}{-\infty-\infty} = \frac{-\infty}{-\infty}$ F.I.

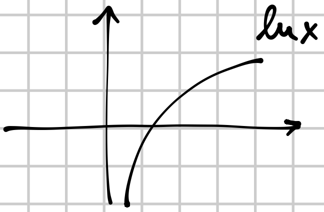
$$= \lim_{x\to-\infty}\frac{x\left(8+\frac{2}{x}\right)}{x-\sqrt{x^2\left(1-\frac{3}{x^2}\right)}} = \lim_{x\to-\infty}\frac{x\left(8+\frac{2}{x}\right)}{x-|x|\sqrt{1-\frac{3}{x^2}}} =$$

$|x| = -x$ perché $x\to-\infty$

$$= \lim_{x\to-\infty}\frac{x\left(8+\frac{2}{x}\right)}{x+x\sqrt{1-\frac{3}{x^2}}} = \lim_{x\to-\infty}\frac{\cancel{x}\left(8+\overset{0}{\frac{2}{x}}\right)}{\cancel{x}\left(1+\sqrt{1-\underset{0}{\frac{3}{x^2}}}\right)} = \frac{8}{2} = 4$$

287 $\displaystyle\lim_{x\to -2} e^{\frac{x^2-4}{x+2}} = \lim_{x\to -2} e^{\frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}} = e^{-2-2} = e^{-4}$

319 $\displaystyle\lim_{x\to 3^-} \frac{\ln(3-x)}{x^3-x^2-6x} = \frac{\ln 0^+}{27-9-18} = \frac{-\infty}{0^-} = +\infty$



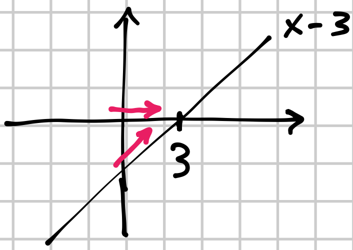
$x^3-x^2-6x =$

$= x(x^2-x-6) =$

$= \underset{\substack{\downarrow\\ 3}}{x}\;\underset{\substack{\downarrow\\ 0^-}}{(x-3)}\;\underset{\substack{\downarrow\\ 5}}{(x+2)}$

$\frac{0}{\infty} = 0$

$\frac{\infty}{0} = \infty$



323 $\displaystyle\lim_{x\to -1} \log_9 \frac{\sqrt[3]{x}+1}{x+1} = \log_9 \frac{-1+1}{-1+1} = \log_9\left(\frac{0}{0}\right)$ F.I.

$\displaystyle = \lim_{x\to -1} \log_9 \frac{\cancel{\sqrt[3]{x}+1}}{\cancel{(\sqrt[3]{x}+1)}(\sqrt[3]{x^2}-\sqrt[3]{x}+1)} = \log_9 \frac{1}{1+1+1} =$

$= \log_9 \frac{1}{3} = \log_9 3^{-1} =$

$x+1 = (\sqrt[3]{x})^3 + 1^3$

$A^3+B^3 = (A+B)(A^2-AB+B^2)$

$= -\log_9 3 = -\frac{1}{2}$

322 $\lim_{x\to 2} \log_2 \frac{\sqrt{x^2+12}-4}{3x^2-4x-4} = (*)$ [− 4]

$$\frac{\sqrt{x^2+12}-4}{(3x+2)(x-2)} \cdot \frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4} = \frac{x^2+12-16}{(3x+2)(x-2)(\sqrt{x^2+12}+4)} =$$

	3	−4	−4
2		6	4
	3	2	//

$$= \frac{x^2-4}{(3x+2)(x-2)(\sqrt{x^2+12}+4)} =$$

$$= \frac{\cancel{(x-2)}(x+2)}{(3x+2)\cancel{(x-2)}(\sqrt{x^2+12}+4)} \xrightarrow[x\to 2]{} \frac{\cancel{4}}{8\cdot \cancel{8}_2} =$$

$$= \frac{1}{16}$$

$$(*) = \log_2 \frac{1}{16} = \log_2 2^{-4} = -4$$

317 $\lim_{x\to 0^+} \frac{2\ln x - 3}{12 - \ln x} = \frac{-\infty}{+\infty}$ F.I. [− 2]

$$= \lim_{x\to 0^+} \frac{\cancel{\ln x}\left(2 - \frac{3}{\ln x}\right)}{\cancel{\ln x}\left(\frac{12}{\ln x} - 1\right)} = \frac{2}{-1} = -2$$

$\frac{3}{\ln x} \to 0$, $\frac{12}{\ln x} \to 0$