3/11/2020

$0^0 \quad 1^\infty \quad \infty^0$

$[f(x)]^{g(x)} = e^{g(x)\ln f(x)}$

254 $\lim_{x\to+\infty} (x+1)^{\frac{1}{\ln x}} = \infty^0$ F.I. $[e]$

$= \lim_{x\to+\infty} e^{\ln (x+1)^{\frac{1}{\ln x}}} = \lim_{x\to+\infty} e^{\frac{1}{\ln x}\cdot \ln(x+1)} = e^1 = e$

$\frac{1}{\ln x}\cdot \ln(x+1) \to 1$

A PARTE

$\lim_{x\to+\infty} \frac{\ln(x+1)}{\ln x} = \lim_{x\to+\infty} \frac{\ln\left(x\left(1+\frac{1}{x}\right)\right)}{\ln x} = \lim_{x\to+\infty} \frac{\ln x + \ln\left(1+\frac{1}{x}\right)}{\ln x} =$

$\ln(a\cdot b) = \ln a + \ln b$

$= \lim_{x\to+\infty} \frac{\cancel{\ln x}\left[1+\frac{\ln\left(1+\frac{1}{x}\right)}{\ln x}\right]}{\cancel{\ln x}} =$

$= \lim_{x\to+\infty} \left[1+\frac{\ln\left(1+\frac{1}{x}\right)}{\ln x}\right] = 1$

$\ln\left(1+\frac{1}{x}\right) \to 0$, $\ln x \to +\infty$

$\frac{0}{\infty} = 0$

256 $\lim_{x\to+\infty} x^{\frac{1}{1+\ln x}} = \infty^0$ F.I. $[e]$

$= \lim_{x\to+\infty} e^{\ln x^{\frac{1}{1+\ln x}}} = \lim_{x\to+\infty} e^{\frac{1}{1+\ln x}\cdot \ln x} = \lim_{x\to+\infty} e^{\frac{\ln x}{1+\ln x}} = e^1 = e$

$\frac{\ln x}{1+\ln x} \to 1$

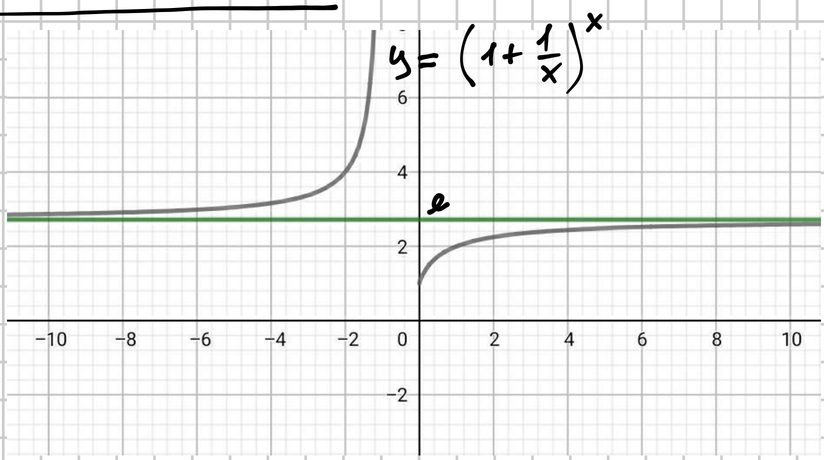
A PARTE

$\lim_{x\to+\infty} \frac{\ln x}{1+\ln x} = \lim_{x\to+\infty} \frac{\cancel{\ln x}}{\cancel{\ln x}\left(\frac{1}{\ln x}+1\right)} = 1$

$\frac{1}{\ln x} \to 0$

LIMITI NOTEVOLI

$$\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$$



- $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

- $$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

- $$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

- $$\lim_{x \to 0} \frac{\ln(x+1)}{x} = 1$$

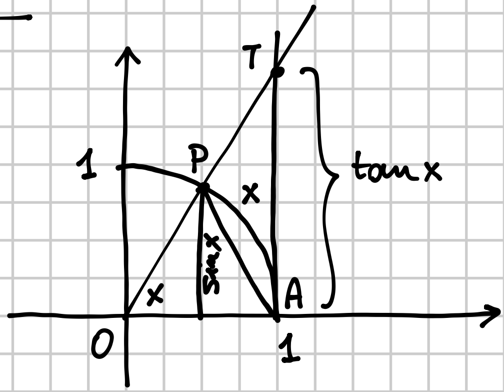
- $$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

- $$\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha$$

  $\alpha \in \mathbb{R}$

DIMOSTRAZIONI

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



x in rad.

$\overset{\frown}{AP} = x$

$$\mathcal{A}_{\triangle OPA} < \mathcal{A}_{\overset{\frown}{OPA}} < \mathcal{A}_{\triangle OTA}$$

area triangolo — area settore circolare

$0 < x < \frac{\pi}{2}$

$r = 1$

AREA DEL SETTORE CIRCOLARE (x RADIANTI)

$$\pi r^2 : 2\pi = \mathcal{A}_{SETT.} : x$$

AREA CERCHIO — ANGOLO AL CENTRO — AREA SETTORE — ANGOLO AL CENTRO DEL SETTORE

$$\mathcal{A}_{SETT.} = \frac{x \cdot \cancel{\pi} r^2}{2\cancel{\pi}} = \frac{1}{2} \underbrace{(r\,x)}_{\text{"BASE"}} \cdot \underbrace{r}_{\text{"ALTEZZA"}}$$

$$\Downarrow$$

$$\frac{1}{2}\sin x < \frac{1}{2}x < \frac{1}{2}\tan x$$

$$\sin x < x < \tan x$$

$$\frac{1}{\tan x} < \frac{1}{x} < \frac{1}{\sin x}$$

$$\frac{\sin x}{\tan x} < \frac{\sin x}{x} < 1 \implies \cos x < \frac{\sin x}{x} < 1$$

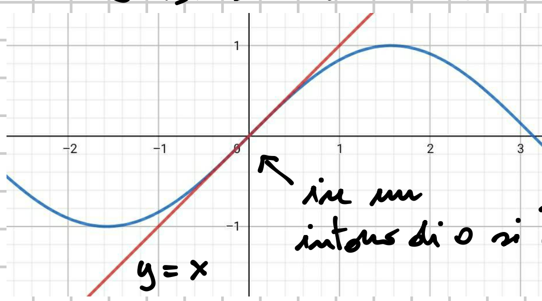
$\cos x \to 1$, $1 \to 1$ per $x \to 0^+$

$\frac{\sin x}{x} \to 1$ per il TH. DEI CARABINIERI

con lo stesso procedimento si vede che la stessa cosa vale per $x \to 0^-$.

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(x RADIANTI)



$y = \sin x$

in un intorno di 0 si "confondono"

356 $\lim_{x\to 0} \frac{x^2+x}{2x+\sin x} = \frac{0}{0}$ $\left[\frac{1}{3}\right]$

$$= \lim_{x\to 0} \frac{\cancel{x}(\overset{\to 0}{x}+1)}{\cancel{x}\left(2+\frac{\sin x}{x}\right)} = \frac{1}{2+1} = \frac{1}{3}$$

$$\frac{\sin x}{x} \to 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{\overbrace{1-\cos^2 x}^{\sin^2 x}}{x(1+\cos x)} =$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1+\cos x} = 1 \cdot \frac{0}{2} = 0$$

$$\frac{\sin x}{x} \to 1 \qquad \cos x \to 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\frac{\sin x}{x} \cdot \frac{\sin x}{x}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)} = \frac{1}{2}$$

$$\frac{\sin^2 x}{x^2} \to 1 \qquad \cos x \to 1$$

357 $\lim_{x \to 0} \frac{2x^2}{1 - \cos x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{2}{\frac{1-\cos x}{x^2}} = \frac{2}{\frac{1}{2}} = 4$$

$\frac{1}{2}$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{x \to 0} \frac{1}{x} \cdot \ln(1+x) = \lim_{x \to 0} \ln(1+x)^{\frac{1}{x}} =$$

$$= \lim_{t \to \infty} \ln\left(1+\frac{1}{t}\right)^t = \ln(e) = 1$$

$\frac{1}{x} = t \Rightarrow x = \frac{1}{t}$

$x \to 0 \Rightarrow t \to \infty$

SOSTITUZIONE DI VARIABILE

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim_{t\to 0} \frac{t}{\ln(t+1)} = \lim_{t\to 0} \frac{1}{\frac{\ln(t+1)}{t}} = 1$$

$\frac{\ln(t+1)}{t} \downarrow 1$

$\boxed{e^x - 1 = t}$ SOST. VARIABILE

$\downarrow$

$e^x = t + 1$

$\downarrow$

$x = \ln(t+1)$

$x \to 0 \Rightarrow t \to 0$

$$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha$$

$\alpha \in \mathbb{R}$

$$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \lim_{x\to 0} \frac{e^{\alpha\cdot\ln(1+x)} - 1}{x} \cdot \frac{\alpha\cdot\ln(1+x)}{\alpha\cdot\ln(1+x)} =$$

$$= \lim_{x\to 0} \frac{e^{\alpha\ln(1+x)} - 1}{\alpha\ln(1+x)} \cdot \lim_{x\to 0} \frac{\alpha\cdot\ln(1+x)}{x} = \lim_{t\to 0} \frac{e^t - 1}{t} \cdot \lim_{x\to 0} \alpha \frac{\ln(1+x)}{x}$$

$\frac{e^t - 1}{t} \to 1$, $\frac{\ln(1+x)}{x} \to 1$

$t = \alpha\ln(1+x)$

$x \to 0 \Rightarrow t \to 0$

$$= 1 \cdot \alpha \cdot 1 = \alpha$$

416 $\lim_{x\to 0} \frac{\sqrt[6]{1+3x}-1}{x} = \frac{0}{0}$ F.I.

$$= \lim_{x\to 0} \frac{(1+3x)^{\frac{1}{6}}-1}{x} \cdot \frac{3}{3} = \lim_{x\to 0} \frac{(1+3x)^{\frac{1}{6}}-1}{3x} \cdot 3 =$$

uguali e tendono a 0
per $x \to 0$
(è come sostituire $t=3x$)

$$= \frac{1}{6} \cdot 3 = \frac{1}{2}$$

435 $\lim_{x\to 0} \frac{(1+2x)^5-1}{5x} = \frac{0}{0}$

$$= \lim_{x\to 0} \frac{(1+2x)^5-1}{5x} \cdot \frac{2}{2} = \lim_{x\to 0} \frac{(1+2x)^5-1}{2x} \cdot \frac{2}{5} =$$

5

$$= 5 \cdot \frac{2}{5} = 2$$

427 $\lim_{x\to 4} \frac{\ln(x-3)}{x-4} = \frac{\ln(1)}{0} = \frac{0}{0}$ F.I.

$$= \lim_{t\to 0} \frac{\ln(t+4-3)}{t} = \lim_{t\to 0} \frac{\ln(t+1)}{t} = 1$$

$x-4=t \Rightarrow x=t+4$

$x\to 4 \Rightarrow t\to 0$

428 $\lim_{x \to 0} \dfrac{e^{3x} - e^{2x}}{x^2 - 3x} = \dfrac{0}{0}$ F.I.

$$= \lim_{x \to 0} \frac{\overbrace{e^{2x}}^{1}\,\overbrace{(e^x - 1)}^{1}}{x\underbrace{(x-3)}_{-3}} = \frac{1}{-3} = -\frac{1}{3}$$