5/11/2020

430 $\lim_{x\to\pm\infty}\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}} = 1^{\infty}$ F.I. $\left[\frac{1}{\sqrt{e}}\right]$

$$= \lim_{x\to\pm\infty} e^{\ln\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}}} = \lim_{x\to\pm\infty} e^{\frac{x}{2}\ln\left(\frac{3x-1}{3x+2}\right)} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

A PARTE

$$\frac{x}{2}\ln\left(\frac{3x-1}{3x+2}\right) = \frac{x}{2}\ln\left(\frac{3x+2-3}{3x+2}\right) = \frac{x}{2}\ln\left(\frac{3x+2}{3x+2}-\frac{3}{3x+2}\right) =$$

$$= \frac{x}{2}\ln\left(1-\frac{3}{3x+2}\right) = \frac{-t-\frac{2}{3}}{2}\ln\left(1+\frac{1}{t}\right) =$$

$-\frac{3}{3x+2} = \frac{1}{t}$ SOSTITUZIONE DI VARIABILE

$\Downarrow$

$-\frac{3x+2}{3} = t$

$\Downarrow$

$3x+2 = -3t$

$3x = -3t-2$

$x = -t-\frac{2}{3}$

$t \to \mp\infty$ per $x \to \pm\infty$

$$= -\frac{t}{2}\cdot\ln\left(1+\frac{1}{t}\right) - \frac{1}{3}\ln\left(1+\frac{1}{t}\right)$$

$$= -\frac{1}{2}\ln\underbrace{\left(1+\frac{1}{t}\right)^{t}}_{\to e} - \frac{1}{3}\ln\left(1+\frac{1}{t}\right)$$

$$-\frac{1}{2}\ln(e) = -\frac{1}{2} \qquad 0$$

$$\downarrow$$

$-\frac{1}{2}$ per $x \to \pm\infty$

447 $\lim_{x\to 0} \frac{\sqrt[6]{1-x}-1}{e^{2x}-1} = \frac{0}{0}$ F.I. $\left[-\frac{1}{12}\right]$

$$\lim_{x\to 0} \frac{(1+x)^{\alpha}-1}{x} = \alpha$$

$\alpha \in \mathbb{R}$

$$\lim_{x\to 0} \frac{e^{x}-1}{x} = 1$$

$$\lim_{x\to L} \frac{e^{f(x)}-1}{f(x)} = 1$$

se $f(x) \to 0$ per $x \to L$

$$= \lim_{x\to 0} \frac{\sqrt[6]{1-x}-1}{e^{2x}-1} \cdot \frac{2x}{2x} =$$

($\frac{2x}{e^{2x}-1} \to 1$)

$$= \lim_{x\to 0} \frac{(1-x)^{\frac{1}{6}}-1}{2x} \cdot \frac{2x}{e^{2x}-1} =$$

$$= \lim_{x\to 0} \frac{(1+(-x))^{1/6}-1}{(-2)(-x)} \cdot \frac{2x}{e^{2x}-1} = \frac{1}{-2} \cdot \frac{1}{6} \cdot 1 = -\frac{1}{12}$$

($\frac{(1+(-x))^{1/6}-1}{-x} \to \frac{1}{6}$, $\frac{2x}{e^{2x}-1} \to 1$)

posso vederlo con una sostituzione

$t = -x$

$t \to 0$ per $x \to 0$

$$\frac{(1+t)^{1/6}-1}{t} \to \frac{1}{6}$$ per $t \to 0$

VALE

$$\lim_{x\to L} \frac{(1+f(x))^{\alpha}-1}{f(x)} = \alpha$$

se $\lim_{x\to L} f(x) = 0$

446 $\lim_{x\to 0} \frac{\ln(x+1)}{\sin 2x + \sin x} = \frac{0}{0}$ F.I. $\left[\frac{1}{3}\right]$

$$= \lim_{x\to 0} \frac{\frac{\ln(x+1)}{x}}{\frac{\sin 2x + \sin x}{x}} = \lim_{x\to 0} \frac{\frac{\ln(x+1)}{x}}{2\cdot\frac{\sin 2x}{2x} + \frac{\sin x}{x}} = \frac{1}{2\cdot 1 + 1} = \frac{1}{3}$$

($\frac{\ln(x+1)}{x} \to 1$, $\frac{\sin 2x}{2x} \to 1$, $\frac{\sin x}{x} \to 1$)

442 $\lim_{x\to 0} \frac{e^{2+x^2} - e^2}{1-\cos^2 x} = \frac{0}{0}$ F.I. $[e^2]$

$$= \lim_{x\to 0} \frac{e^2(e^{x^2}-1)}{1-\cos^2 x} = \lim_{x\to 0} \frac{e^2(e^{x^2}-1)}{\sin^2 x} \cdot \frac{x^2}{x^2} = e^2$$

($\frac{e^{x^2}-1}{x^2} \to 1$, $\frac{x^2}{\sin^2 x} \to 1$)

$\frac{\sin x^2}{x^2} \to 1$ per $x \to 0$

$\frac{\sin^2 x}{x^2} = \frac{\sin x}{x} \cdot \frac{\sin x}{x} \to 1$ per $x \to 0$ ($\frac{\sin x}{x} \to 1$)

$\frac{x^2}{\sin x^2} = \frac{1}{\frac{\sin x^2}{x^2}} \to 1$

($\frac{\sin x^2}{x^2} \to 1$ per $x \to 0$)

444 $\lim_{x\to 0} \frac{\tan x}{e^{\sin x} - \cos x} = \frac{0}{0}$ F.I. [1]

$$= \lim_{x\to 0} \frac{\frac{\tan x}{x}}{\frac{e^{\sin x} - \cos x}{x}} = \lim_{x\to 0} \frac{\frac{\tan x}{x}}{\frac{e^{\sin x} - 1 + 1 - \cos x}{x}} =$$

$$\frac{\tan x}{x} = \frac{\sin x}{(\cos x)\, x} \to 1$$

per $x \to 0$ ($\frac{\sin x}{x} \to 1$, $\cos x \to 1$)

$$= \lim_{x\to 0} \frac{\frac{\tan x}{x}}{\frac{e^{\sin x} - 1}{x} + \frac{1 - \cos x}{x}} =$$

$$= \lim_{x\to 0} \frac{\frac{\tan x}{x}}{\frac{e^{\sin x} - 1}{x} \cdot \frac{\sin x}{\sin x} + \frac{1 - \cos x}{x}} = \frac{1}{1 \cdot 1 + 0} = 1$$

($\frac{\tan x}{x} \to 1$, $\frac{e^{\sin x} - 1}{\sin x} \to 1$, $\frac{\sin x}{x} \to 1$, $\frac{1-\cos x}{x} \to 0$)

IN GENERALE:

$$\lim_{x\to 0} \frac{\tan x}{x} = 1 \qquad \lim_{x\to 0} \frac{\arctan x}{x} = 1 \qquad \lim_{x\to 0} \frac{\arcsin x}{x} = 1$$