9/11/2020

507 $\lim_{x\to+\infty} \frac{\ln x}{\ln(x+2)} = \frac{\infty}{\infty}$ F.I. [1]

$$= \lim_{x\to+\infty} \frac{\ln x}{\ln\left(x\left(1+\frac{2}{x}\right)\right)} = \lim_{x\to+\infty} \frac{\ln x}{\ln x + \ln\left(1+\frac{2}{x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{\ln x}}{\cancel{\ln x}\left[1 + \underbrace{\frac{\ln\left(1+\frac{2}{x}\right)}{\ln x}}_{\frac{0}{\infty}=0}\right]} = 1$$

535 $\lim_{x\to+\infty} \frac{(1-x)^{2x}}{(1+x^2)^x} = \frac{\infty}{\infty}$ F.I. $\left[\frac{1}{e^2}\right]$

$$= \lim_{x\to+\infty} \left(\frac{(1-x)^2}{1+x^2}\right)^x = \lim_{x\to+\infty} \left(\frac{1+x^2-2x}{1+x^2}\right)^x =$$

$$= \lim_{x\to+\infty} \left(1 - \frac{2x}{x^2+1}\right)^x = \lim_{x\to+\infty} \left(1 - \frac{1}{\frac{x^2+1}{2x}}\right)^x =$$

$$= \lim_{x\to+\infty} \underbrace{\left[\left(1 + \frac{1}{-\frac{x^2+1}{2x}}\right)^{-\frac{x^2+1}{2x}}\right]}_{e}^{\underbrace{-\frac{2x^2}{x^2+1}}_{\to -2}} = e^{-2} = \frac{1}{e^2}$$

431 $\lim_{x\to+\infty} x[\ln(x^2+4)-2\ln x]$ = [0]

$$= \lim_{x\to+\infty} x\left[\ln(x^2+4) - \ln x^2\right] =$$

$$= \lim_{x\to+\infty} x\left[\ln\left(\frac{x^2+4}{x^2}\right)\right] = \lim_{x\to+\infty} x\left[\ln\left(1+\frac{4}{x^2}\right)\right] =$$

$$= \lim_{x\to+\infty} \frac{\ln\left(1+\frac{4}{x^2}\right)}{\frac{1}{x}\cdot\frac{4x}{4x}} = \lim_{x\to+\infty} \frac{\ln\left(1+\frac{4}{x^2}\right)}{\frac{4}{x^2}\cdot\frac{x}{4}} = \frac{1}{+\infty} = 0$$

$\to 1$ $\quad \frac{x}{4} \to +\infty$

usato il limite notevole

$$\lim_{t\to 0} \frac{\ln(1+t)}{t} = 1$$

con $t = \frac{4}{x^2}$

541 $\displaystyle\lim_{x\to 0}\frac{e^{\sin 2x}-e^{\sin x}}{\tan x} = \frac{0}{0}$ [1]

$$= \lim_{x\to 0}\frac{e^{\sin 2x}-e^{\sin x}}{\tan x} = \lim_{x\to 0}\frac{e^{\sin x}\left(e^{\sin 2x-\sin x}-1\right)}{\tan x} =$$

$$= \lim_{x\to 0}\frac{e^{\sin x}\left(e^{2\cos x\sin x-\sin x}-1\right)}{\tan x} =$$

$$\lim_{x\to 0}\frac{\underbrace{e^{\sin x}}_{\to 1}\left(e^{\sin x(2\cos x-1)}-1\right)}{\frac{x}{x}\cdot\tan x\cdot\sin x(2\cos x-1)}\cdot\sin x\,\underbrace{(2\cos x-1)}_{\to 1} = 1$$

se $f(x)\to 0$ per $x\to x_0$ senza annullarsi in un intorno di x_0, escluso il punto x_0 stesso (cioè nei casi più frequenti), valgono le seguenti equivalenze per $x\to x_0$

$\sin f(x)\sim f(x)$	$\tan f(x)\sim f(x)$	$1-\cos f(x)\sim \frac{1}{2}f^2(x)$
$e^{f(x)}-1\sim f(x)$	$[1+f(x)]^\alpha-1\sim \alpha f(x)\quad(\alpha\in\mathbb{R})$	$\ln(1+f(x))\sim f(x)$

539 $\displaystyle\lim_{x\to 0}\frac{\sqrt[4]{1+x^3}-1}{x^3-x^4} = \frac{0}{0}$ $\left[\frac{1}{4}\right]$

$$\frac{(1+x^3)^{\frac{1}{4}}-1}{x^3-x^4}\sim\frac{\frac{1}{4}\cancel{x^3}}{\cancel{x^3}}=\frac{1}{4}\quad\text{quindi}\quad\lim_{x\to 0}\frac{\sqrt[4]{1+x^3}-1}{x^3-x^4}=\frac{1}{4}$$

per $x\to 0$

$x^3-x^4\sim x^3$ per $x\to 0$ perché $\displaystyle\lim_{x\to 0}\frac{x^3-x^4}{x^3}=\lim_{x\to 0}(1-x)=1$