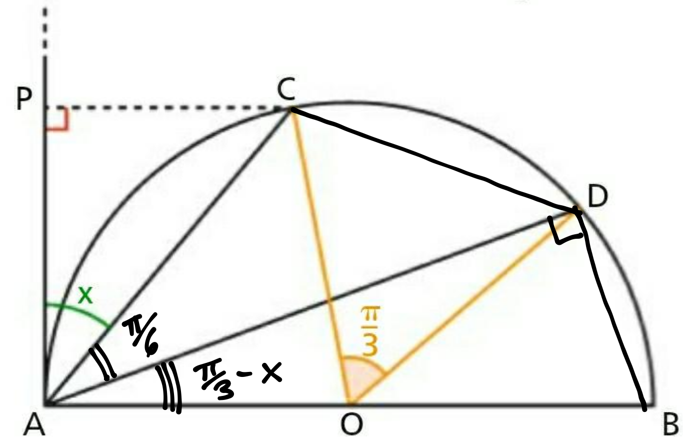
10/11/2020

571 Nella semicirconferenza di diametro $\overline{AB} = 2r$ in figura, esprimi in funzione dell'angolo x il rapporto tra $\overline{AP} \cdot \overline{CD}$ e l'area del triangolo ACD.



Calcola quindi il limite di tale rapporto al tendere di C ad A. [4]

$C\hat{O}D$ è un triangolo equilatero

$\overline{CD} = r$

$C\hat{A}D = \frac{1}{2}\frac{\pi}{3} = \frac{\pi}{6}$ perché angolo alla circonferenza che corrisponde all'angolo al centro $C\hat{O}D$

$D\hat{A}B = \frac{\pi}{2} - x - \frac{\pi}{6} = \frac{\pi}{3} - x \qquad 0 < x < \frac{\pi}{3}$

Da calcolare $\lim_{x \to 0^+} \frac{\overline{AP} \cdot \overline{CD}}{\mathcal{A}_{ACD}}$

$\overline{AP} = \overline{AC} \cdot \cos x \qquad \mathcal{A}_{ACD} = \frac{1}{2}\overline{AC} \cdot \overline{AD} \cdot \sin\frac{\pi}{6} = \frac{1}{4}\overline{AC} \cdot \overline{AD}$

$\overline{AD} = 2r \cdot \cos\left(\frac{\pi}{3} - x\right)$

$$\frac{\overline{AP} \cdot \overline{CD}}{\mathcal{A}_{ACD}} = \frac{\cancel{\overline{AC}} \cdot \cos x \cdot \cancel{r}}{\cancel{\frac{1}{4}}_{2} \cancel{\overline{AC}} \cdot \cancel{2r} \cos\left(\frac{\pi}{3} - x\right)} = \frac{2\cos x}{\cos\left(\frac{\pi}{3} - x\right)}$$

$$\lim_{x \to 0^+} \frac{2\cos x}{\cos\left(\frac{\pi}{3} - x\right)} = \frac{2 \cdot 1}{\cos\frac{\pi}{3}} = \frac{2}{\frac{1}{2}} = \boxed{4}$$

REPRISE

541 $\lim_{x \to 0} \dfrac{e^{\sin 2x} - e^{\sin x}}{\tan x} =$

$$= \lim_{x \to 0} \frac{e^{\sin 2x} - 1 + 1 - e^{\sin x}}{\tan x} =$$

$$= \lim_{x \to 0} \frac{e^{\sin 2x} - 1}{\tan x} - \lim_{x \to 0} \frac{e^{\sin x} - 1}{\tan x} =$$

$$= \lim_{x \to 0} \frac{\sin 2x}{x} - \lim_{x \to 0} \frac{\sin x}{x} =$$

$$= \lim_{x \to 0} \frac{2\cancel{x}}{\cancel{x}} - 1 = 2 - 1 = 1$$

ATTENZIONE!!

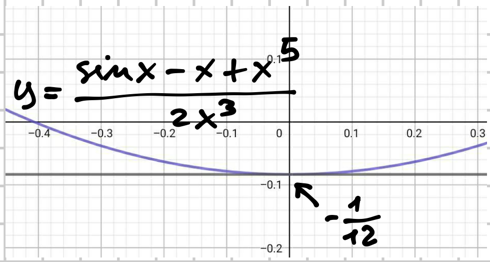
1) $\lim_{x \to 0} \dfrac{1 - \cos x}{x^2} \neq \lim_{x \to 0} \dfrac{1 - 1}{x^2} = \lim_{x \to 0} 0 = 0$

$\cos x \sim 1$ per $x \to 0$

LA SOSTITUZIONE DI UNA FUNZIONE CON UNA SUA ASINTOTICA NON FUNZIONA, PERCHÉ RISULTA 0 DOPO LA SOSTITUZIONE

$1-1$

2) $\underbrace{\lim_{x \to 0} \dfrac{\sin x - x + x^5}{2x^3}}_{= -\frac{1}{12}} \neq \lim_{x \to 0} \dfrac{\cancel{x} - \cancel{x} + x^5}{2x^3} = \lim_{x \to 0} \dfrac{x^2}{2} = 0$



426 $\lim_{x\to+\infty}\left(x\ln\frac{3x+1}{3x}\right) = \infty \cdot 0$ F.I. $\left[\frac{1}{3}\right]$

$$= \lim_{x\to+\infty} x \cdot \ln\left(1+\frac{1}{3x}\right) = \frac{1}{3}$$

$$x \cdot \ln\left(1+\frac{1}{3x}\right) \sim x \cdot \frac{1}{3x} = \frac{1}{3}$$

$$\ln(1+f(x)) \sim f(x)$$

per $f(x)\to 0$

435 $\lim_{x\to 0}\frac{(1+2x)^5-1}{5x} = 2$ $[2]$

$$(1+f(x))^\alpha - 1 \sim \alpha f(x)$$

per $f(x)\to 0$

$$\frac{(1+2x)^5-1}{5x} \sim \frac{\cancel{5}\cdot 2\cancel{x}}{\cancel{5}\cancel{x}} = 2 \quad \text{per } x\to 0$$

445 $\lim_{x \to 0} \frac{e^{\sin 4x} - 1}{\ln(1 + \tan x)} = 4$ [4]

$$\frac{e^{\sin 4x} - 1}{\ln(1 + \tan x)} \sim \frac{\sin 4x}{\tan x} \sim \frac{4x}{x} = 4$$

446 $\lim_{x \to 0} \frac{\ln(x + 1)}{\sin 2x + \sin x} =$ $\left[\frac{1}{3}\right]$

$$= \lim_{x \to 0} \frac{\ln(x+1)}{2 \sin x \cos x + \sin x} = \lim_{x \to 0} \frac{\ln(x+1)}{\sin x (2\cos x + 1)} =$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x} \cdot \underbrace{(2\underbrace{\cos x}_{1} + 1)}_{3}} = \frac{1}{3}$$

447 $\lim\limits_{x \to 0} \dfrac{\sqrt[6]{1-x}-1}{e^{2x}-1} = \dfrac{0}{0}$ F.I. $\left[-\dfrac{1}{12}\right]$

$$= \lim_{x \to 0} \frac{\frac{1}{6}(-x)}{2x} = \frac{-\frac{1}{6}}{2} = -\frac{1}{12}$$

440 $\lim\limits_{x \to 0} \dfrac{\cos x - \ln(1+x) - 1}{2x} = \dfrac{0}{0}$ F.I. $\left[-\dfrac{1}{2}\right]$

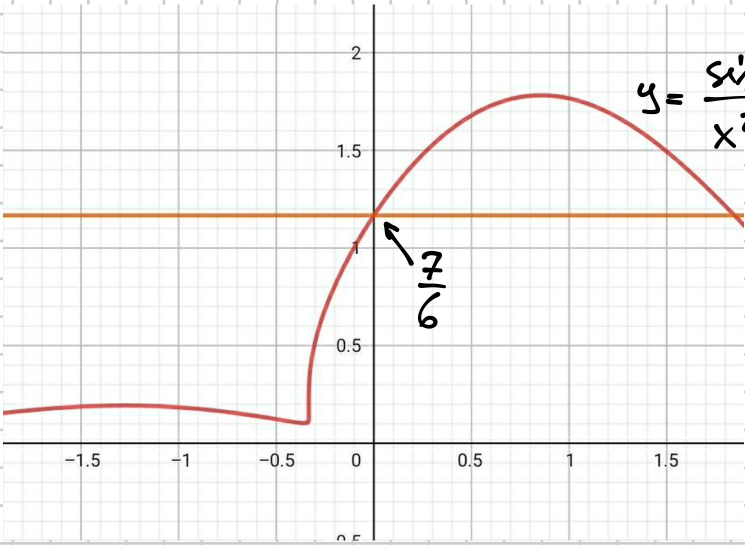
$$= \lim_{x \to 0} \frac{\cos x - 1}{2x} - \lim_{x \to 0} \frac{\ln(1+x)}{2x} =$$

$$= -\lim_{x \to 0} \frac{1-\cos x}{2x} - \lim_{x \to 0} \frac{\ln(1+x)}{2x} = -0 - \frac{1}{2} = -\frac{1}{2}$$

0 1

450 $\lim_{x\to 0} \frac{\sin x(1-\cos x)}{x^2(\sqrt[7]{1+3x}-1)} = \frac{7}{6}$ $\left[\frac{7}{6}\right]$

$$\frac{\sin x\,(1-\cos x)}{x^2\left((1+3x)^{1/7}-1\right)} \sim \frac{\cancel{x}\cdot\frac{1}{2}\cancel{x^2}}{\cancel{x^2}\cdot\frac{1}{7}\cdot 3\cancel{x}} = \frac{\frac{1}{2}}{\frac{3}{7}} = \frac{1}{2}\cdot\frac{7}{3} = \frac{7}{6}$$



521 $\lim_{x\to 2} \frac{3e^2\sin(x-2)}{4e^x-(2e)^2} =$ $\left[\frac{3}{4}\right]$

$$= \lim_{x\to 2} \frac{3e^2\sin(x-2)}{4e^x-4e^2} = \lim_{x\to 2} \frac{3\cancel{e^2}\sin(x-2)}{4\cancel{e^2}(e^{x-2}-1)} =$$

$$= \lim_{x\to 2} \frac{3\cancel{(x-2)}}{4\cancel{(x-2)}} = \frac{3}{4}$$

525 $\lim_{x \to -1} (x+2)^{\frac{2}{x+1}} = 1^{\infty}$ [e^2]

$$= \lim_{x\to -1} e^{\ln (x+2)^{\frac{2}{x+1}}} = \lim_{x\to -1} e^{\frac{2}{x+1}\cdot \ln(x+2)} = e^2$$

A PARTE

$$\lim_{x\to -1} \frac{2\ln(x+2)}{x+1} = 2 \lim_{x\to -1} \frac{\ln(\overbrace{(x+1)}^{f(x)}+1)}{\underbrace{x+1}_{f(x)}} = 2\cdot 1 = 2$$

$\to 1$

$f(x) \to 0$ per $x \to -1$

con la sost. di variabile

$$\lim_{x\to -1} \frac{\ln((x+1)+1)}{x+1} = \lim_{t\to 0} \frac{\ln(t+1)}{t} = 1$$

$t = x+1 \qquad x \to -1 \Rightarrow t \to 0$