4/12/2020

188 $y = -\dfrac{\sqrt[3]{x^2} + 4x}{\sqrt{x}}$ $\left[y' = \dfrac{-(12\sqrt[3]{x} + 1)}{6\sqrt[6]{x^5}} \right]$

dom $f =]0, +\infty[$

$x > 0$

$$f(x) = -\frac{x^{\frac{2}{3}} + 4x}{x^{\frac{1}{2}}} = -\frac{x^{\frac{2}{3}}}{x^{\frac{1}{2}}} - 4\frac{x}{x^{\frac{1}{2}}} =$$

$$= -x^{\frac{2}{3}-\frac{1}{2}} - 4x^{1-\frac{1}{2}} = -x^{\frac{4-3}{6}} - 4x^{\frac{1}{2}} =$$

$$= -x^{\frac{1}{6}} - 4x^{\frac{1}{2}}$$

$$f'(x) = -\frac{1}{6}x^{\frac{1}{6}-1} - 4 \cdot \frac{1}{2}x^{\frac{1}{2}-1} = -\frac{1}{6}x^{-\frac{5}{6}} - 2x^{-\frac{1}{2}} =$$

$$= -\frac{1}{6\sqrt[6]{x^5}} - \frac{2}{\underbrace{\sqrt{x}}_{\sqrt[6]{x^3}}} = \frac{-1 - 12\sqrt[6]{x^2}}{6\sqrt[6]{x^5}} =$$

$$= -\frac{1 + 12\sqrt[3]{x}}{6\sqrt[6]{x^5}}$$

189 $y = \sqrt{\sqrt{x}} - \ln\frac{1}{x^2} + e^4$ $\left[y' = \frac{1}{4\sqrt[4]{x^3}} + \frac{2}{x}\right]$

$$y = \sqrt[4]{x} - \ln x^{-2} + e^4 =$$
$$= x^{\frac{1}{4}} + 2\ln x + e^4$$

DERIVATA DI $\ln x$ $(x>0)$

$$(\ln x)' = \lim_{h\to 0} \frac{\ln(x+h) - \ln x}{h} =$$
$$= \lim_{h\to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h} = \lim_{h\to 0} \frac{\ln\left(1+\frac{h}{x}\right)}{\frac{h}{x}\cdot x} = \frac{1}{x}$$

1

$$y = x^{\frac{1}{4}} + 2\ln x + e^4$$ COSTANTE

$$y' = \frac{1}{4}x^{\frac{1}{4}-1} + 2\cdot\frac{1}{x} = \frac{1}{4}x^{-\frac{3}{4}} + \frac{2}{x} = \frac{1}{4\sqrt[4]{x^3}} + \frac{2}{x}$$

Calcolare la derivata:

$y = \log_a x$

$$y = \log_a x = \frac{\ln x}{\ln a}$$

$$y' = \frac{dy}{dx} = \frac{1}{\ln a}\cdot(\ln x)' = \frac{1}{x\ln a}$$

Calcolare la derivata di $y = a^x$

$$y = a^x = e^{\ln a^x} = e^{x \ln a}$$ ← COMPOSIZIONE DI e^x CON $x \ln a$

$$y = f(g(x)) \Rightarrow y' = f'(g(x)) \cdot g'(x)$$

$$y' = (a^x)' = (e^{x \ln a})' = \overbrace{e^{x \ln a}}^{a^x} \cdot \ln a = a^x \cdot \ln a$$

$f(x) = e^x \qquad f'(x) = e^x$

$g(x) = x \ln a \qquad g'(x) = \ln a$

1067 $y = \ln \frac{x}{x+3}$

1) DOMINIO

$$\frac{x}{x+3} > 0$$

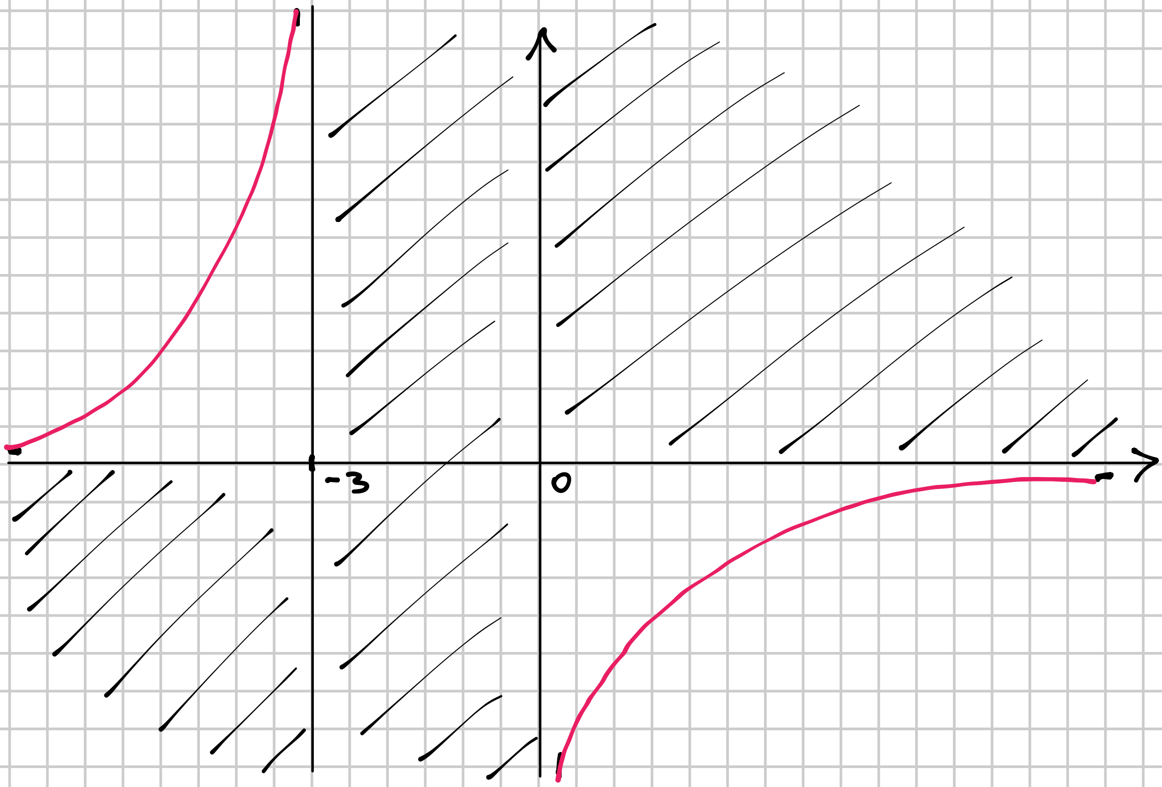
$x > 0$

$x + 3 > 0 \Rightarrow x > -3$

		-3		0	
x	$-$		$-$		$+$
$x+3$	$-$		$+$		$+$
	$+$	$\times$	$-$	ϕ	$+$

$x < -3 \vee x > 0$

$$D =]-\infty, -3[\cup]0, +\infty[$$



2) INT. ASSI

$$\ln\left(\frac{x}{x+3}\right) = 0 \qquad \frac{x}{x+3} = 1 \Rightarrow x = x + 3 \quad \text{IMPOSS.}$$

NON CI SONO INTERSEZIONI

3) SEGNO

$$\ln \frac{x}{x+3} > 0 \quad \Rightarrow \quad \frac{x}{x+3} > 1 \qquad \frac{x}{x+3} - 1 > 0$$

$$\frac{\cancel{x} - \cancel{x} - 3}{x+3} > 0$$

$$-\frac{3}{x+3} > 0$$

$$\frac{3}{x+3} < 0$$

$$\Downarrow$$

$$x+3 < 0 \qquad x < -3$$

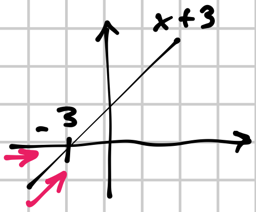
4) LIMITI $\quad D =]-\infty, -3[\cup]0, +\infty[$

$$\lim_{x \to -\infty} \ln \frac{x}{x+3} = \ln(1) = 0$$

$y = 0$ ASINTOTO ORIZZ. per $x \to -\infty$

$$\lim_{x \to -3^-} \ln \frac{x}{\underbrace{x+3}_{0^-}} = \ln \frac{-3}{0^-} = \ln(+\infty) = +\infty$$

$x = -3$ ASINTOTO VERTICALE



$$\lim_{x \to 0^+} \ln \frac{x}{x+3} = \ln 0^+ = -\infty$$

$x = 0$ ASINTOTO VERTICALE

$$\lim_{x \to +\infty} \ln \frac{x}{x+3} = \ln(1) = 0$$

$y = 0$ ASINTOTO ORIZZ. per $x \to +\infty$