

14/12/2020

DERIVATA DEL PRODOTTO DI DUE FUNZIONI

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x) \quad (\text{FORMULA DI LEIBNIZ})$$

DIMOSTRAZIONE

$$[f(x) \cdot g(x)]' = \lim_{h \rightarrow 0}$$

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{g(x+h)[f(x+h) - f(x)] + f(x)[g(x+h) - g(x)]}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \cdot g(x+h) + \lim_{h \rightarrow 0} f(x) \cdot \frac{g(x+h) - g(x)}{h} =$$

\downarrow \downarrow

$f'(x)$ $g'(x)$

$$= f'(x)g(x) + f(x) \cdot g'(x)$$

ESEMPI

$$1) y = e^x \cdot \sin x \quad y' = (e^x)' \cdot \sin x + e^x \cdot (\sin x)' = e^x \cdot \sin x + e^x \cos x \\ = e^x (\sin x + \cos x)$$

$$2) y = x^2 \cdot \cos x \quad y' = 2x \cdot \cos x + x^2 \cdot (-\sin x) \\ = 2x \cos x - x^2 \sin x$$

$$3) \quad y = 5x^3 \Rightarrow$$

↓
 applic
 la formula
 di Leibniz

$$y' = \underbrace{(5)}_0 \cdot x^3 + 5 \cdot (x^3)' = 0 \cdot x^3 + 5 \cdot 3x^2 =$$

$$= 15x^2$$

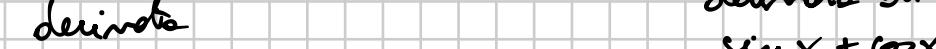
$$y' = 15x^2$$

↓

È coerente con le regole delle costante

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$$y' = \frac{1}{2} \left[e^x (\sin x + \cos x) + e^x \cdot (\cos x - \sin x) \right] -$$


 derivate di
 e^x

derivate di
 $\sin x + \cos x$

$$-\left[e^x \cos x + e^x (-\sin x)\right] =$$

$$= \frac{1}{2} [e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x] - e^x \cos x + e^x \sin x =$$

$$= e^x \cancel{\cos x} - e^x \cancel{\cos x} + e^x \sin x = e^x \sin x$$

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$$y = x \cdot e^x \cdot \ln x = (x \cdot e^x) \cdot \ln x$$

$$\begin{aligned}
 y' &= (x \cdot e^x)' \cdot \ln x + x \cdot e^x \cdot (\ln x)' = \\
 &= (e^x + x \cdot e^x) \cdot \ln x + \cancel{x} \cdot e^x \cdot \frac{1}{\cancel{x}} = \\
 &= e^x \ln x + x \cdot e^x \ln x + e^x = \\
 &= e^x (\ln x + x \ln x + 1)
 \end{aligned}$$

IN GENERALE

$$\begin{aligned}
 [f(x) g(x) h(x)]' &= (f(x) g(x))' h(x) + f(x) g(x) h'(x) = \\
 &= [f'(x) g(x) + f(x) g'(x)] h(x) + f(x) g(x) h'(x) = \\
 &= f'(x) g(x) h(x) + f(x) g'(x) h(x) + f(x) g(x) h'(x)
 \end{aligned}$$