14/12/2020

DERIVATA DEL PRODOTTO DI DUE FUNZIONI

$$[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$ (FORMULA DI LEIBNIZ)

DIMOSTRAZIONE

$$[f(x)\cdot g(x)]' = \lim_{h\to 0} \frac{\overbrace{f(x+h)\,g(x+h) - f(x)\,g(x)}^{fg\,(x+h)}}{h} =$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h} =$$

$$= \lim_{h\to 0} \frac{g(x+h)\,[f(x+h)-f(x)] + f(x)\,[g(x+h)-g(x)]}{h} =$$

$$= \lim_{h\to 0} \underbrace{\frac{f(x+h)-f(x)}{h}}_{\downarrow\, f'(x)} \cdot \underbrace{g(x+h)}_{\downarrow\, g(x)} + \lim_{h\to 0} f(x)\cdot \underbrace{\frac{g(x+h)-g(x)}{h}}_{\downarrow\, g'(x)} =$$

$$= f'(x)\,g(x) + f(x)\cdot g'(x)$$

ESEMPI

1) $y = e^x\cdot \sin x$ $\quad y' = (e^x)'\cdot \sin x + e^x\cdot(\sin x)' = e^x\cdot \sin x + e^x \cos x$

$= e^x(\sin x + \cos x)$

2) $y = x^2\cdot \cos x$ $\quad y' = 2x\cdot \cos x + x^2\cdot(-\sin x)$

$= 2x\cos x - x^2 \sin x$

3) $y = 5x^3 \Rightarrow$ applico la formula di Leibniz $\qquad y' = \underbrace{(5)'}_{0}x^3 + 5\cdot(x^3)' = 0\cdot x^3 + 5\cdot 3x^2 =$

$= 15x^2$

$\Downarrow$

$y' = 15x^2$

$\downarrow$

È coerente con la regola della costante

208 $y = \frac{1}{2}e^x(\sin x + \cos x) - e^x \cos x$

$$y' = \frac{1}{2}\left[e^x(\sin x + \cos x) + e^x\cdot\underbrace{(\cos x - \sin x)}_{\text{derivata di } \sin x + \cos x}\right] -$$

derivata di e^x

$$-\left[e^x\cos x + e^x(-\sin x)\right] =$$

$$= \frac{1}{2}\left[\cancel{e^x\sin x} + e^x\cos x + e^x\cos x - \cancel{e^x\sin x}\right] - e^x\cos x + e^x\sin x =$$

$$= \cancel{e^x\cos x} - \cancel{e^x\cos x} + e^x\sin x = e^x\sin x$$

214 $y = x \cdot e^x \cdot \ln x = (x \cdot e^x) \cdot \ln x$

$$y' = (xe^x)' \cdot \ln x + xe^x \cdot (\ln x)' =$$

$$= (e^x + xe^x) \cdot \ln x + \cancel{x} e^x \cdot \frac{1}{\cancel{x}} =$$

$$= e^x \ln x + xe^x \ln x + e^x =$$

$$= e^x (\ln x + x \ln x + 1)$$

IN GENERALE

$$[f(x)\, g(x)\, h(x)]' = (f(x)\, g(x))' h(x) + f(x)\, g(x)\, h'(x) =$$

$$= [f'(x)\, g(x) + f(x)\, g'(x)]\, h(x) + f(x)\, g(x)\, h'(x) =$$

$$= f'(x)\, g(x)\, h(x) + f(x)\, g'(x)\, h(x) + f(x)\, g(x)\, h'(x)$$