21/12/2020

Data la funzione $y = f(x)$, calcola la derivata della funzione inversa $x = g(y)$ nel punto y_0 indicato a fianco.

450 $f(x) = 4x + \ln x$, $y_0 = 4$. $\left[g'(4) = \frac{1}{5}\right]$

452 $f(x) = e^{x-1} + x$, $y_0 = 2$. $\left[g'(2) = \frac{1}{2}\right]$

450] $f(x) = 4x + \ln x$

$$(f^{-1})'(4) = ? \qquad (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

OSSERVAZIONE

$y = 4x + \ln x$ è BIETTIVA? sì, perché è strettamente crescente

$$x_1 < x_2 \Rightarrow \begin{array}{l} 4x_1 < 4x_2 \\ \ln x_1 < \ln x_2 \end{array} \Rightarrow \underbrace{4x_1 + \ln x_1}_{f(x_1)} < \underbrace{4x_2 + \ln x_2}_{f(x_2)}$$

$f'(x) = 4 + \frac{1}{x}$ devo calcolare $f^{-1}(4)$

$$4 = 4x + \ln x$$
$$\Downarrow$$
$$x = 1$$

$f^{-1}(4) = 1$

$$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{4 + \frac{1}{1}} = \frac{1}{5}$$

452)

$$f(x) = e^{x-1} + x \qquad f'(x) = e^{x-1} + 1$$

$$(f^{-1})'(2) = ? \qquad f^{-1}(2) = ?$$

$$\Downarrow$$

$$e^{x-1} + x = 2 \Rightarrow \underbrace{x = 1}_{f^{-1}(2)}$$

$$(f^{-1})'(2) = \frac{1}{f'(\underbrace{f^{-1}(2)}_{1})} = \frac{1}{f'(1)} = \frac{1}{e^{1-1}+1} = \frac{1}{2}$$

341 $y = (2x^2)^{x^2}$ $\qquad [y' = 2x(2x^2)^{x^2}(\ln 2x^2 + 1)]$

$$f(x)^{g(x)} = e^{g(x)\ln f(x)}$$

$$y = (2x^2)^{x^2} = e^{\ln(2x^2)^{x^2}} = e^{x^2 \ln(2x^2)}$$

$$y' = \underbrace{e^{x^2\ln(2x^2)}}_{(2x^2)^{x^2}} \cdot [x^2 \ln(2x^2)]' =$$

$$= (2x^2)^{x^2} \cdot \left[2x \cdot \ln(2x^2) + \cancel{x^2} \cdot \frac{1}{\cancel{2x^2}} \cdot \cancel{4}^{2}x\right] =$$

$$= (2x^2)^{x^2} \cdot 2x \left(\ln(2x^2) + 1\right) =$$

$$= 2x\,(2x^2)^{x^2}\left(\ln(2x^2) + 1\right)$$

527 $y = x^2 e^{x^2} + 2$ $\quad [y' = 2xe^{x^2}(1 + x^2)]$

$$y' = 2x \cdot e^{x^2} + x^2 \cdot e^{x^2} \cdot 2x = 2xe^{x^2}(1 + x^2)$$

542 $y = \arctan\sqrt{x^2 - 1}$ $\quad \left[y' = \dfrac{1}{x\sqrt{x^2 - 1}}\right]$

$$y' = \frac{1}{1 + (x^2 - 1)} \cdot \left(\sqrt{x^2 - 1}\right)' = \frac{1}{x^{\cancel{2}}} \cdot \frac{1}{\cancel{2}\sqrt{x^2 - 1}} \cdot \cancel{2}\cancel{x} =$$

$$= \frac{1}{x\sqrt{x^2 - 1}}$$

571 $y = 4\arcsin\dfrac{x}{2} + x\sqrt{4 - x^2}$ $\quad \left[y' = 2\sqrt{4 - x^2}\right]$

$$y' = \overset{2}{\cancel{4}} \cdot \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \frac{1}{\cancel{2}} + 1 \cdot \sqrt{4 - x^2} + x \cdot \frac{1}{\cancel{2}\sqrt{4 - x^2}} \cdot (-\cancel{2}x) =$$

$$= \frac{2}{\sqrt{1 - \frac{x^2}{4}}} + \sqrt{4 - x^2} - \frac{x^2}{\sqrt{4 - x^2}} =$$

$$= \frac{2}{\frac{\sqrt{4 - x^2}}{2}} + \sqrt{4 - x^2} - \frac{x^2}{\sqrt{4 - x^2}} = \frac{4 + 4 - x^2 - x^2}{\sqrt{4 - x^2}}$$

$$= \frac{2\cancel{(4 - x^2)}}{\sqrt{\cancel{4 - x^2}}} \cdot \frac{\sqrt{4 - x^2}}{\sqrt{\cancel{4 - x^2}}} = \boxed{2\sqrt{4 - x^2}}$$

569 $y = \dfrac{1-2x^2}{\sqrt{1-x^2}}$ $\left[y' = \dfrac{2x^3-3x}{\sqrt{(1-x^2)^3}} \right]$

$$y' = \frac{-4x\sqrt{1-x^2} - (1-2x^2)\cdot \dfrac{1}{\cancel{2}\sqrt{1-x^2}}\cdot(-\cancel{2}x)}{1-x^2} =$$

$$= \frac{\dfrac{-4x(1-x^2)+x(1-2x^2)}{\sqrt{1-x^2}}}{1-x^2} =$$

$$= \frac{-4x+4x^3+x-2x^3}{(1-x^2)\sqrt{1-x^2}} =$$

$$= \boxed{\frac{2x^3-3x}{(1-x^2)\sqrt{1-x^2}}}$$