22/12/2021

150 $\dfrac{2}{x^3+8} - \dfrac{1}{x^3-8} = 0$ $\quad [2\sqrt[3]{3}]$

$(x+2)(x^2-2x+4) \quad (x-2)(x^2+2x+4)$

$\Delta < 0$

NON SI SCOMPONGONO (SONO SEMPRE POSITIVI, CIOÈ $\forall x$)

C.E.

$x \neq \pm 2$

$$\frac{2(x-2)(x^2+2x+4) - (x+2)(x^2-2x+4)}{(x+2)(x-2)(x^2-2x+4)(x^2+2x+4)} = 0$$

$$2(x^3-8) - (x^3+8) = 0$$

$$2x^3 - 16 - x^3 - 8 = 0$$

$$x^3 = 24 \qquad x = \sqrt[3]{24} = \sqrt[3]{3 \cdot 2^3} = 2\sqrt[3]{3}$$

CURIOSITÀ

$$x^6 - 1 = (x^2-1)(x^4+x^2+1) = (x-1)(x+1)\underbrace{(x^4+x^2+1)}_{\text{come scomporlo?}} = (*)$$

$$x^4+x^2+1 = x^4+x^2+x^2+1-x^2 =$$

$$= x^4+2x^2+1-x^2 = (x^2+1)^2 - x^2 =$$

$$= [(x^2+1)-x][(x^2+1)+x] = (x^2-x+1)(x^2+x+1)$$

$$(*) = (x-1)(x+1)(x^2-x+1)(x^2+x+1)$$

(si poteva inizialmente anche scomporre $x^6-1 = (x^3-1)(x^3+1) =$
$= (x-1)(x^2+x+1)(x+1)(x^2-x+1)$)

161 $x^3 + 4 + \dfrac{8}{x^3 - 2} = \dfrac{x^3 + 6}{x^3 - 2}$ $\left[-\sqrt[3]{3}\right]$

C.E. $x \neq \sqrt[3]{2}$

$$\frac{(x^3+4)(x^3-2)+8}{x^3-2} = \frac{x^3+6}{x^3-2}$$

$$x^6 - 2x^3 + 4x^3 - 8 + 8 - x^3 - 6 = 0$$

$$x^6 + x^3 - 6 = 0 \qquad (x^3+3)(x^3-2) = 0$$

$x^3 - 2 = 0 \quad x = \sqrt[3]{2}$ N.A.

$x^3 + 3 = 0$

$\Downarrow$

$x^3 = -3$

$\boxed{x = -\sqrt[3]{3}}$

232 $x^3 - 2x^2 - 7x + 12 = 0$

RUFFINI

$\pm 1 \ \pm 2 \ \pm 3 \ \pm 4 \ \pm 6 \ \pm 12$

$1 \mapsto 1^3 - 2\cdot 1^2 - 7\cdot 1 + 12 = 1 - 2 - 7 + 12 = 4 \neq 0$ ☹

$-1 \mapsto (-1)^3 - 2\cdot(-1)^2 - 7\cdot(-1) + 12 = -1 - 2 + 7 + 12 \neq 0$ ☹

$2 \mapsto 8 - 8 - 14 + 12 \neq 0$ 😑

$-2 \mapsto -8 - 8 + 14 + 12 \neq 0$ 😨

$3 \mapsto 27 - 18 - 21 + 12 = 0$ ☺ OK!

	1	−2	−7	12
3		3	3	−12
	1	1	−4	//

$(x^2 + x - 4)(x - 3) = 0$

$x^2 + x - 4 = 0 \qquad \Delta = 1 + 16 = 17 \qquad x = \dfrac{-1 \pm \sqrt{17}}{2}$

$$\boxed{x = 3 \ \vee \ x = \frac{-1 \pm \sqrt{17}}{2}}$$

209 $(2x+1)^3 - 2x^2 - 5x - 2 = 0$ $\left[-1; -\frac{1}{2}; \frac{1}{4}\right]$

$8x^3 + 12x^2 + 6x + 1 - 2x^2 - 5x - 2 = 0$

$8x^3 + 10x^2 + x - 1 = 0$ ± 1

$1 \mapsto 8 + 10 + 1 - 1 \neq 0$

$-1 \mapsto -8 + 10 - 1 - 1 = 0$ OK!!

	8	10	1	−1
−1		−8	−2	1
	8	2	−1	//

$(x+1)(8x^2 + 2x - 1) = 0$

$8x^2 + 2x - 1 = 0$ $\frac{\Delta}{4} = 1 + 8 = 9$

$x = \frac{-1 \pm 3}{8} = \begin{cases} -\frac{1}{2} \\ \frac{2}{8} = \frac{1}{4} \end{cases}$

$$\boxed{x = -1 \ \vee \ x = -\frac{1}{2} \ \vee \ x = \frac{1}{4}}$$