10/2/2022

1) $3x^2 + x + 2 > 0$ $\quad \Delta = 1 - 4\cdot3\cdot2 = 1 - 24 =$

$= -23 < 0$

INS. SOLUZIONE $\quad S = \mathbb{R} \qquad \forall x \in \mathbb{R}$

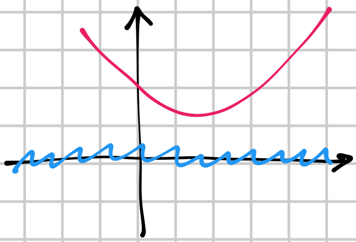
$\forall x$

2) $2x^2 - x + 7 < 0$ $\quad \Delta = 1 - 4\cdot2\cdot7 = 1 - 56 = -55 < 0$

INS. SOLUZIONE $\quad S = \emptyset \qquad \nexists x$ IMPOSSIBILE

<u>INTERPRETAZIONE GRAFICA</u>

$y = ax^2 + bx + c \qquad a > 0 \qquad \Delta < 0$



CASO $\Delta = 0$

$a > 0$ $\quad ax^2 + bx + c > 0$

$\Delta = 0$ $\quad a(x - x_1)^2 > 0$

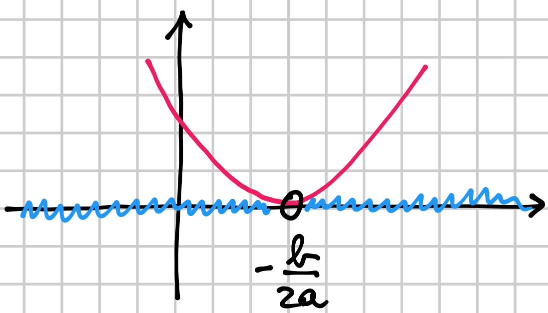
$\Downarrow$ $\quad\quad \Downarrow$

$x_1 = x_2 = \frac{-b}{2a}$ $\quad a\left(x + \frac{b}{2a}\right)^2 > 0$

per quali x questa relazione è vera?

Per tutti gli $x \neq -\frac{b}{2a}$

$S = \mathbb{R} \setminus \left\{-\frac{b}{2a}\right\}$ $\quad \forall x \neq -\frac{b}{2a}$ $\quad x \neq -\frac{b}{2a}$



Se fosse $ax^2 + bx + c \geq 0$, l'insieme soluzione sarebbe $\mathbb{R}$

$a > 0$
$\Delta = 0$

Se fosse $ax^2 + bx + c < 0$, l'insieme soluzione sarebbe $\emptyset$

Se fosse $ax^2 + bx + c \leq 0$, l'insieme soluzione sarebbe $\left\{-\frac{b}{2a}\right\}$

$x = -\frac{b}{2a}$

ESEMPI

1) $3x^2 - 2\sqrt{3}\,x + 1 > 0$ $\qquad \Delta = (-2\sqrt{3})^2 - 4\cdot 3 = 0$

$$x_1 = x_2 = -\frac{b}{2a} = -\frac{-2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$

$$\forall x \neq \frac{\sqrt{3}}{3}$$

2) $x^2 - 2x + 1 \leq 0$ $\qquad \frac{\Delta}{4} = 1 - 1 = 0$

$$-\frac{b}{2a} = 1$$

$$x = 1$$

$$S = \{x \in \mathbb{R} \mid x^2 - 2x + 1 \geq 0\} = \{x \in \mathbb{R} \mid x = 1\} = \{1\}$$

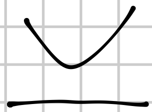
230 $\dfrac{x+2}{4} - \dfrac{x+1}{2} > x^2 + 1$

$$\frac{x+2-2(x+1)}{\cancel{4}} > \frac{4(x^2+1)}{\cancel{4}}$$

$$x + \cancel{2} - 2x - \cancel{2} > 4x^2 + 4$$

$$-4x^2 - x - 4 > 0$$

$$4x^2 + x + 4 < 0 \qquad \Delta = 1 - 4 \cdot 4 \cdot 4 < 0$$



IMPOSSIBILE

$\emptyset$

245 $x\sqrt{2} - \dfrac{x-3}{\sqrt{2}-1} \leq (x+\sqrt{2})^2 + 3\sqrt{2} - x - 2$

$$\frac{x\sqrt{2}(\sqrt{2}-1) - x + 3}{\cancel{\sqrt{2}-1}} \leq \frac{(x^2+2+2\sqrt{2}x)(\sqrt{2}-1) + (\sqrt{2}-1)(3\sqrt{2}-x-2)}{\cancel{\sqrt{2}-1}}$$

$$\cancel{2x} - \cancel{\sqrt{2}x} - \cancel{x} + 3 \leq \sqrt{2}x^2 - x^2 + \cancel{2\sqrt{2}} - \cancel{2} + 4x - 2\sqrt{2}x + 6 - \cancel{\sqrt{2}x} - \cancel{2\sqrt{2}} - 3\sqrt{2} + \cancel{x} + \cancel{2}$$

$$x^2 - \sqrt{2}x^2 - 4x + 2\sqrt{2}x + 3 - 6 + 3\sqrt{2} \leq 0$$

$$(1-\sqrt{2})x^2 - 2(2-\sqrt{2})x - 3 + 3\sqrt{2} \leq 0$$

$a < 0$ cambiare i segni!

$$(\sqrt{2}-1)x^2 + 2(2-\sqrt{2})x + 3 - 3\sqrt{2} \geq 0$$

$$\left(\sqrt{2}-1\right)x^2+2\left(2-\sqrt{2}\right)x+3-3\sqrt{2}\geq 0$$

$$\frac{\Delta}{4}=\left(2-\sqrt{2}\right)^2-\left(\sqrt{2}-1\right)\left(3-3\sqrt{2}\right)=4+2-4\sqrt{2}-\left(3\sqrt{2}-6-3+3\sqrt{2}\right)=$$

$$=6-4\sqrt{2}+9-6\sqrt{2}=6+9-10\sqrt{2}=15-10\sqrt{2}=5\left(3-2\sqrt{2}\right)=$$

$$=\left[\sqrt{5}\left(\sqrt{2}-1\right)\right]^2$$

$$x_{1,2}=\frac{-\left(2-\sqrt{2}\right)\pm\sqrt{5}\left(\sqrt{2}-1\right)}{\sqrt{2}-1}$$

$$x_1=\frac{-2+\sqrt{2}-\sqrt{10}+\sqrt{5}}{\sqrt{2}-1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}+1}=$$

$$=\frac{-2\sqrt{2}\cancel{-2}\cancel{+2}+\sqrt{2}-2\sqrt{5}\cancel{-\sqrt{10}}\cancel{+\sqrt{10}}+\sqrt{5}}{2-1}=-\sqrt{2}-\sqrt{5}$$

$$x_2=\frac{-2+\sqrt{2}+\sqrt{10}-\sqrt{5}}{\sqrt{2}-1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}+1}=$$

$$=\frac{-2\sqrt{2}\cancel{-2}\cancel{+2}+\sqrt{2}+2\sqrt{5}\cancel{+\sqrt{10}}\cancel{-\sqrt{10}}-\sqrt{5}}{2-1}=-\sqrt{2}+\sqrt{5}$$

$$\boxed{x\leq-\sqrt{2}-\sqrt{5}\quad\vee\quad x\geq-\sqrt{2}+\sqrt{5}}$$