2/3/2022

414 $\dfrac{1}{x+\sqrt{2}} < \dfrac{1}{x\sqrt{2}-2}$

$[x < -\sqrt{2} \vee \sqrt{2} < x < 3\sqrt{2}+4]$

$$\frac{1}{x+\sqrt{2}} - \frac{1}{x\sqrt{2}-2} < 0$$

$$\frac{x\sqrt{2}-2-x-\sqrt{2}}{(x+\sqrt{2})(x\sqrt{2}-2)} < 0$$

$$\frac{N}{D} \quad \frac{x(\sqrt{2}-1)-2-\sqrt{2}}{(x+\sqrt{2})(x\sqrt{2}-2)} < 0$$

N $\quad x(\sqrt{2}-1)-2-\sqrt{2} > 0 \qquad x(\sqrt{2}-1) > 2+\sqrt{2}$

$$x > \frac{2+\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2\sqrt{2}+2+2+\sqrt{2}}{2-1} =$$

$$= 3\sqrt{2}+4$$

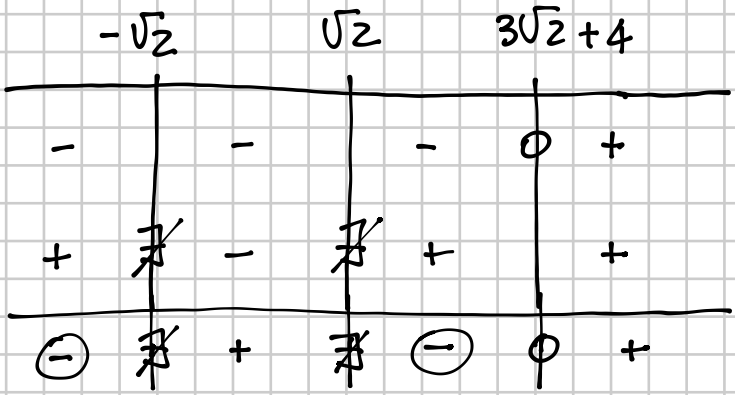
D $\quad (x+\sqrt{2})(x\sqrt{2}-2) > 0 \qquad x < -\sqrt{2} \quad \vee \quad x > \sqrt{2}$

$x_1 = -\sqrt{2}$

$x_2 = \dfrac{2}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

N > 0 $\qquad x > 3\sqrt{2}+4$

D > 0 $\qquad x < -\sqrt{2} \vee x > \sqrt{2}$



$$\boxed{x < -\sqrt{2} \quad \vee \quad \sqrt{2} < x < 3\sqrt{2}+4}$$

387 $\dfrac{(x^2-1)^2(x^2-4x)^3}{2x^2+1} \geq 0$ $[x \leq 0 \vee x = 1 \vee x \geq 4]$

N_1 N_2

D

$N_1 > 0$ $(x^2-1)^2 > 0$ $x^2-1 \neq 0$ $x \neq \pm 1$ (per qualsiasi x tranne per $x = \pm 1$, per cui vale 0)

$N_2 > 0$ $(x^2-4x)^3 > 0$ $x^2-4x > 0$ $x < 0 \vee x > 4$

$x(x-4)$

$x_1 = 0$ $x_2 = 4$

$D > 0$ $2x^2+1 > 0$ $\forall x$

$\Delta = 0^2-4\cdot 2\cdot 1 = -8 < 0$

		-1		0		1		4	
N_1	$x \neq \pm 1$	+	0	+	+	0	+		+
N_2	$x<0 \vee x>4$	+		+	0	−	−	0	+
D	$\forall x$	+		+	+	+	+		+
		⊕	∄	⊕	0	−	∄	−	0	⊕
			≠		≠		≠		≠

$x \leq 0 \vee x = 1 \vee x \geq 4$

391 $\dfrac{(x^2-2\sqrt{2}\,x+2)(5x-x^2-7)}{3x-x^3}\geq 0$

$[-\sqrt{3}<x<0 \vee x=\sqrt{2} \vee x>\sqrt{3}]$

$$\frac{\overset{N_1}{(x^2-2\sqrt{2}x+2)}\,\overset{N_2}{(x^2-5x+7)}}{\underset{D_1}{x}\,\underset{D_2}{(x^2-3)}} \geq 0$$

cambio i segni 2 volte e giro 2 volte il verso della disuguaglianza

$N_1>0 \quad x^2-2\sqrt{2}x+2>0 \quad (x-\sqrt{2})^2>0 \quad x\neq\sqrt{2}$

$N_2>0 \quad x^2-5x+7>0 \quad \forall x$

$\Delta = 25-28=-3<0$

$D_1>0 \quad x>0$

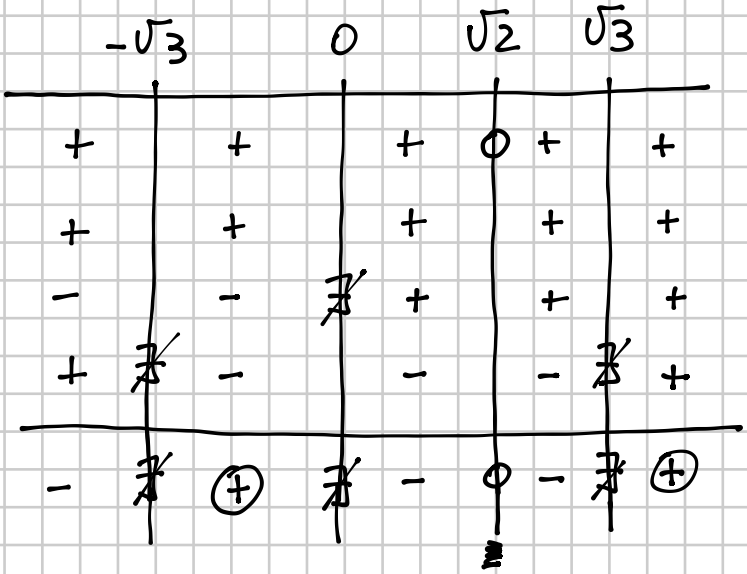
$D_2>0 \quad x^2-3>0 \quad x<-\sqrt{3} \vee x>\sqrt{3}$

$N_1 \quad x\neq\sqrt{2}$

$N_2 \quad \forall x$

$D_1 \quad x>0$

$D_2 \quad x<-\sqrt{3} \vee x>\sqrt{3}$



$$-\sqrt{3}<x<0 \quad \vee \quad x=\sqrt{2} \quad \vee \quad x>\sqrt{3}$$

418 $\dfrac{1}{x^2-6x+9}+\dfrac{1}{3-x}\geq 2$ $\left[2\leq x\leq \dfrac{7}{2}\wedge x\neq 3\right]$

$$\frac{1}{(x-3)^2}+\frac{1}{-(x-3)}-2\geq 0$$

$$\frac{1-(x-3)-2(x-3)^2}{(x-3)^2}\geq 0$$

$$\frac{1-x+3-2x^2-18+12x}{(x-3)^2}\geq 0 \qquad \frac{-2x^2+11x-14}{(x-3)^2}\geq 0$$

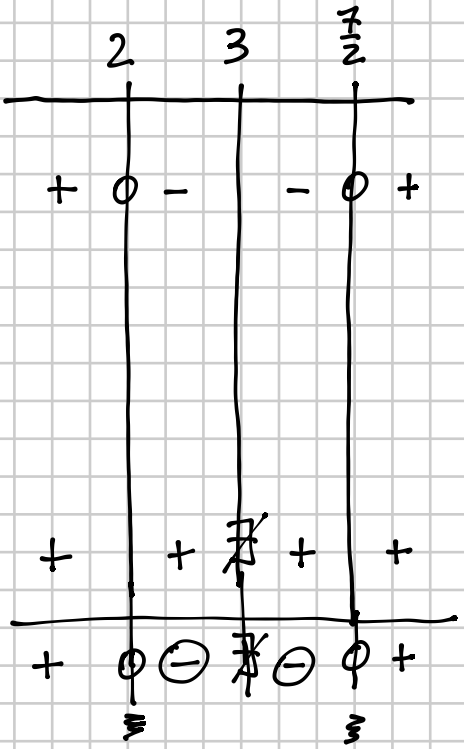
N
$$\frac{2x^2-11x+14}{(x-3)^2}\leq 0$$
D

$N>0 \quad 2x^2-11x+14>0 \quad x<2 \vee x>\frac{7}{2}$

$\Delta = 121-112 = 9$

$x_{1,2}=\frac{11\pm 3}{4}=\begin{cases}2\\ \frac{7}{2}\end{cases}$

$D>0 \quad (x-3)^2>0 \quad x\neq 3$



$$\boxed{2\leq x\leq \frac{7}{2}\wedge x\neq 3}$$

oppure $\quad 2\leq x<3 \vee 3<x\leq \frac{7}{2}$

459

$$\begin{cases} \text{①}\ \dfrac{x-1}{4-x} \le 0 \\ \text{②}\ \dfrac{x-3}{4} < \dfrac{x}{6} \end{cases}$$

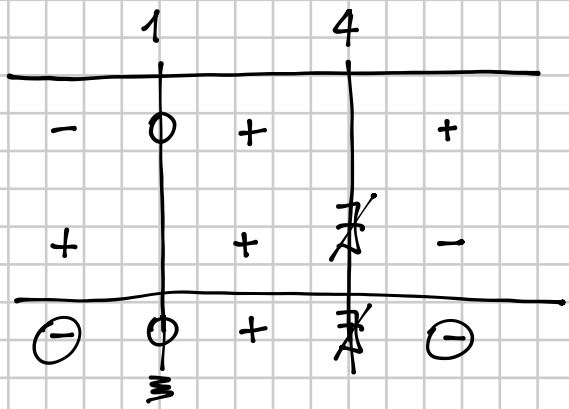
$[x \le 1 \vee 4 < x < 9]$

① $\dfrac{N\ x-1}{D\ 4-x} \le 0$

$N>0 \quad x-1>0 \quad x>1$

$D>0 \quad 4-x>0 \quad -x>-4 \quad x<4$

$x \le 1 \quad \vee \quad x>4$

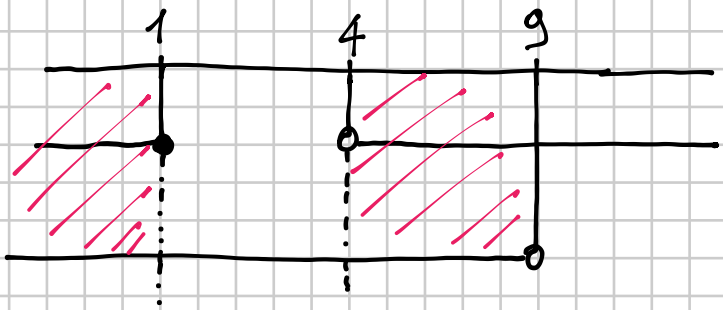


② $\dfrac{x-3}{4} < \dfrac{x}{6} \qquad \dfrac{x-3}{4} - \dfrac{x}{6} < 0 \qquad \dfrac{3x-9-2x}{12} < 0$

$x-9<0 \quad x<9$

① $x \le 1 \quad \vee \quad x>4$

② $x<9$



$x \le 1 \quad \vee \quad 4 < x < 9$