226 $|x^2 - 1| < x + 1$ $\qquad$ $[0 < x < 2]$

del tipo $|f(x)| < g(x)$ $\qquad$ $-g(x) < f(x) < g(x)$

$-(x+1) < x^2 - 1 < x + 1$

$$\begin{cases} x^2 - 1 < x + 1 \\ \\ x^2 - \cancel{1} > -x - \cancel{1} \end{cases}$$

$$\begin{cases} x^2 - x - 2 < 0 \\ \\ x^2 + x > 0 \end{cases} \quad \begin{cases} (x-2)(x+1) < 0 \\ \\ x(x+1) > 0 \end{cases} \quad \begin{cases} -1 < x < 2 \\ \\ x < -1 \;\lor\; x > 0 \end{cases}$$


$$\boxed{0 < x < 2}$$

DISEQUAZIONI DEL TIPO

$$|f(x)| > k \qquad\qquad k \in \mathbb{R}, \ k > 0$$

Bennansitallo

$$|x| > k$$

-k   0   k
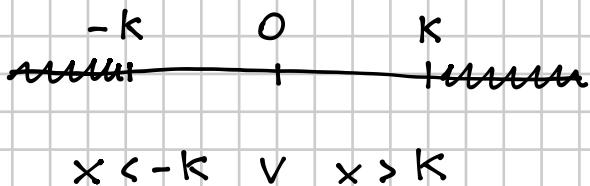
$$x < -k \quad \lor \quad x > k$$

$$|f(x)| > k \quad \Longleftrightarrow \quad f(x) < -k \quad \lor \quad f(x) > k$$

ESEMPIO

$$|x^2 - 1| > 3$$

$$x^2 - 1 < -3 \qquad \lor \qquad x^2 - 1 > 3$$

$$x^2 + 2 < 0 \qquad \lor \qquad x^2 > 4$$

$$\varnothing \qquad\qquad\qquad x < -2 \ \lor \ x > 2$$

$$\Downarrow$$

$$\boxed{x < -2 \ \lor \ x > 2}$$

223 $|3 - x| > \dfrac{1}{2}x$ $[x < 2 \lor x > 6]$

$$3 - x < -\frac{1}{2}x \qquad \lor \qquad 3 - x > \frac{1}{2}x$$

$$-x + \frac{1}{2}x < -3 \qquad \lor \qquad -x - \frac{1}{2}x > -3$$

$$-\frac{1}{2}x < -3 \qquad \lor \qquad -\frac{3}{2}x > -3$$

$$x > 6 \qquad \lor \qquad x < 2 \qquad \Rightarrow \qquad \boxed{x < 2 \ \lor \ x > 6}$$

241 $\boxed{N} \dfrac{x^2 + 4x - 5}{3 - |x|} \geq 0$ $\boxed{D}$ $[-5 \leq x < -3 \vee 1 \leq x < 3]$

$\boxed{N}$

$x^2 + 4x - 5 > 0$ $(x+5)(x-1) > 0$ $x < -5 \vee x > 1$

$\boxed{D}$ $3 - |x| > 0$ $|x| < 3$ $-3 < x < 3$

$\boxed{N}$ $x < -5 \vee x > 1$

$\boxed{D}$ $-3 < x < 3$
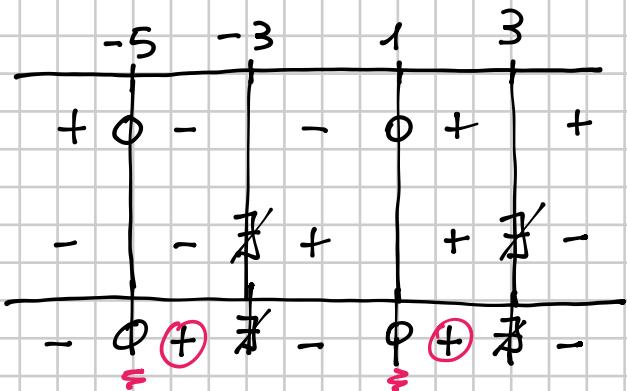
$-5 \leq x < -3 \vee 1 \leq x < 3$

244 $\dfrac{N}{D} \dfrac{|2x-3|-x-1}{4x^2-2x} < 0$ $\left[0 < x < \dfrac{1}{2} \vee \dfrac{2}{3} < x < 4\right]$

N] $|2x-3|-x-1 > 0$ $|2x-3| > x+1$

$2x-3 < -x-1$ $\vee$ $2x-3 > x+1$

$3x < 2$ $\vee$ $x > 4$

$x < \dfrac{2}{3}$ $\vee$ $x > 4$

D] $4x^2-2x > 0$ $2x(2x-1) > 0$ $x < 0$ $\vee$ $x > \dfrac{1}{2}$

N] $x < \dfrac{2}{3}$ $\vee$ $x > 4$

D] $x < 0$ $\vee$ $x > \dfrac{1}{2}$

$0 < x < \dfrac{1}{2}$ $\vee$ $\dfrac{2}{3} < x < 4$
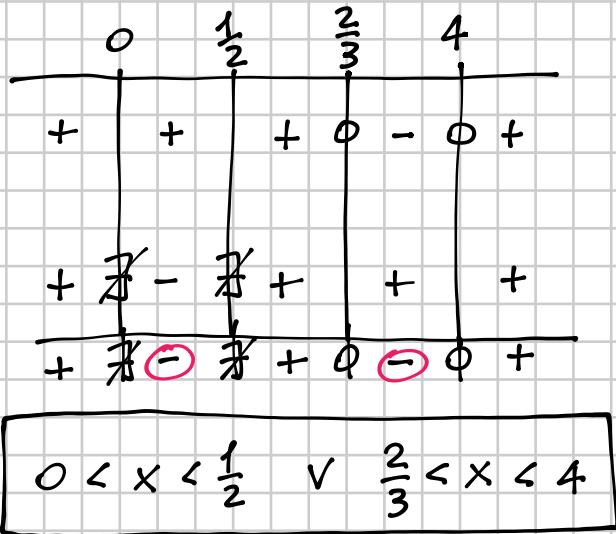
289 $\dfrac{1}{|x|} + \dfrac{2}{|x+1|} = 2$ $\left[-\dfrac{5+\sqrt{17}}{4}; 1 \right]$ C.E.

$$x \neq 0 \qquad x \neq -1$$

$$x > 0$$

$$x + 1 > 0 \qquad x > -1$$

	-1	0	
	$-$	$-$	$+$
	$-$	$+$	$+$

non metto $\leq$ per C.E.

non metto $\geq$ per C.E.
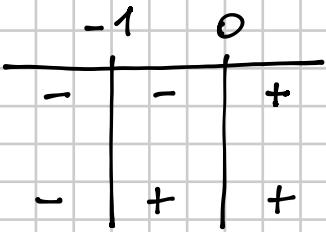
① $\begin{cases} x < -1 \\ -\dfrac{1}{x} - \dfrac{2}{x+1} = 2 \end{cases}$ V   ② $\begin{cases} -1 < x < 0 \\ -\dfrac{1}{x} + \dfrac{2}{x+1} = 2 \end{cases}$ V   ③ $\begin{cases} x > 0 \\ \dfrac{1}{x} + \dfrac{2}{x+1} = 2 \end{cases}$

① $\begin{cases} x < -1 \\ \dfrac{-x-1-2x}{x(x+1)} = \dfrac{2x(x+1)}{x(x+1)} \end{cases}$

$$-3x - 1 = 2x^2 + 2x$$

$$2x^2 + 5x + 1 = 0$$

$$\Delta = 25 - 8 = 17 \qquad x = \dfrac{-5 \pm \sqrt{17}}{4}$$

$$x = \dfrac{-5 - \sqrt{17}}{4}$$

② $\begin{cases} -1 < x < 0 \\ \dfrac{-x-1+2x}{x(x+1)} = \dfrac{2x(x+1)}{x(x+1)} \end{cases}$ $-x - 1 + 2x = 2x^2 + 2x$

$$2x^2 + x + 1 = 0$$

$$\Delta = 1 - 8 < 0 \qquad \varnothing \quad \text{IMPOSSIBILE}$$

③ $\begin{cases} x > 0 \\ \dfrac{1}{x} + \dfrac{2}{x+1} = 2 \end{cases}$

$\dfrac{x+1+2x}{x(x+1)} = \dfrac{2x(x+1)}{x(x+1)}$

$3x + 1 = 2x^2 + 2x$

$2x^2 - x - 1 = 0$

$\Delta = 1 + 8 = 9 \qquad x = \dfrac{1 \pm 3}{4} = \begin{cases} -\dfrac{1}{2} \\ 1 \end{cases}$

$x = 1$

① v ② v ③

$\boxed{x = \dfrac{-5 - \sqrt{17}}{4} \quad v \quad x = 1}$