ESEMPIO

RIPASSO VETTORI

25/9/2021

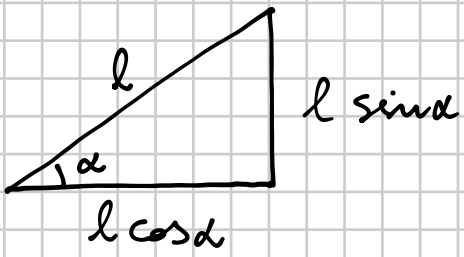
$\vec{v}$ $\quad v = 7$ $\quad \alpha = 35°$

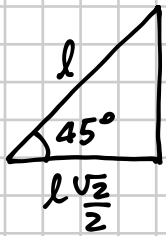
Trovare le componenti cartesiane

$$\vec{v} = \begin{cases} v_x = v \cos 35° = 7 \cdot 0{,}81915\ldots \simeq 5{,}73 \\ v_y = v \sin 35° = 7 \cdot 0{,}5735\ldots \simeq 4{,}02 \end{cases}$$

$$\vec{v} = (5{,}73\ ,\ 4{,}02)$$

$$v = \sqrt{(5{,}73)^2 + (4{,}02)^2} = 6{,}9995\ldots \simeq 7$$

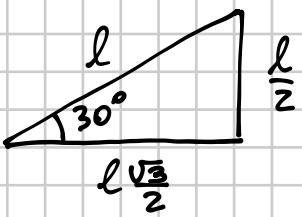


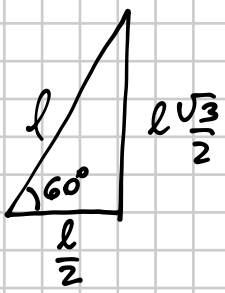


$\cos 0° = 1$ $\qquad$ $\cos 45° = \frac{\sqrt{2}}{2}$

$\cos 90° = 0$ $\qquad$ $\cos 30° = \frac{\sqrt{3}}{2}$

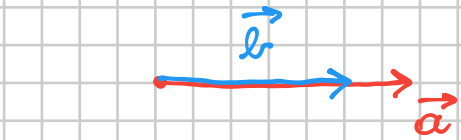
$\cos 180° = -1$ $\qquad$ $\cos 60° = \frac{1}{2}$





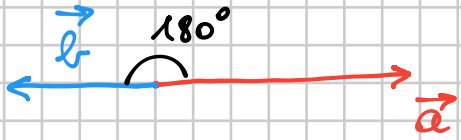
PRODOTTO SCALARE

VETTORI PARALLELI



$$\vec{a} \cdot \vec{b} = a\,b \cos 0° =$$

$$= a\,b$$

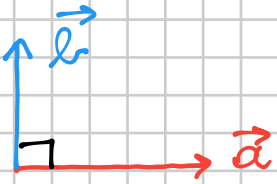


$$\vec{a} \cdot \vec{b} = a\,b \cos 180° =$$

$$= a\,b \cdot (-1) =$$

$$= -a\,b$$

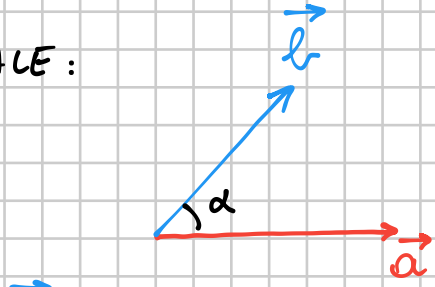
VETTORI PERPENDICOLARI



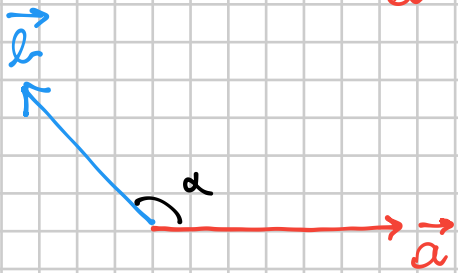
$$\vec{a} \cdot \vec{b} = a\,b \cos 90° =$$

$$= a\,b \cdot 0 = 0$$

IN GENERALE:



$$\vec{a} \cdot \vec{b} = a\,b \cos\alpha > 0$$



$$\vec{a} \cdot \vec{b} = a\,b \cos\alpha < 0$$

PROPRIETÁ DEL PRODOTTO SCALARE

$$\vec{a} \cdot \vec{b} = a b \cos\alpha$$

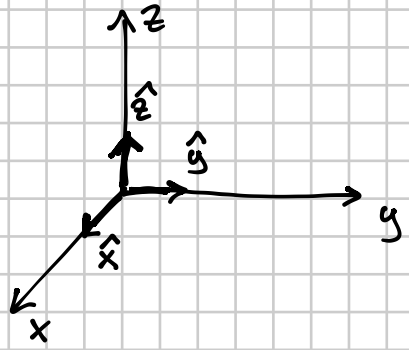
1) PR. COMMUTATIVA

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

2) PR. DISTRIBUTIVA

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

3) VERSORI DEGLI ASSI



$\hat{x} = (1, 0, 0)$

$\hat{y} = (0, 1, 0)$

$\hat{z} = (0, 0, 1)$

$$\hat{x} \cdot \hat{x} = |\hat{x}| \cdot |\hat{x}| \cdot \cos 0° = 1 \cdot 1 \cdot 1 = 1$$

$$\hat{y} \cdot \hat{y} = 1$$

$$\hat{z} \cdot \hat{z} = 1$$

$$\hat{x} \cdot \hat{y} = 0 \qquad \hat{x} \cdot \hat{z} = 0 \qquad \hat{y} \cdot \hat{z} = 0$$

4) PRODOTTO SCALARE DI 2 VETTORI IN COMPONENTI CARTESIANE

$$\vec{a} = (a_x, a_y, a_z) \qquad \vec{b} = (b_x, b_y, b_z)$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{a} = (a_x, a_y) \qquad \vec{b} = (b_x, b_y)$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$

DIMOSTRAZIONE

$$\vec{a} = (a_x, a_y, a_z) = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$$

$$\vec{b} = (b_x, b_y, b_z) = b_x \hat{x} + b_y \hat{y} + b_z \hat{z}$$

$$\vec{a} \cdot \vec{b} = (a_x \hat{x} + a_y \hat{y} + a_z \hat{z}) \cdot (b_x \hat{x} + b_y \hat{y} + b_z \hat{z}) =$$

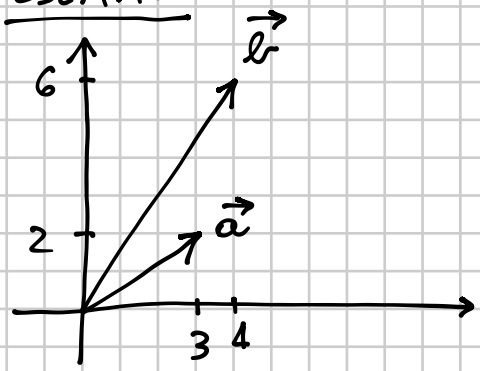
PARALLELI: $a_x \hat{x} \cdot b_x \hat{x}$ — PERPENDICOLARI: $a_x \hat{x} \cdot b_y \hat{y}$ — PERPENDICOLARI: $a_x \hat{x} \cdot b_z \hat{z}$

$$= a_x \hat{x} \cdot b_x \hat{x} + \underbrace{a_x \hat{x} \cdot b_y \hat{y}}_{0} + \underbrace{a_x \hat{x} \cdot b_z \hat{z}}_{0} + \dots$$

$$\dots + a_z \hat{z} \cdot b_z \hat{z} =$$

$$= a_x b_x + a_y b_y + a_z b_z$$

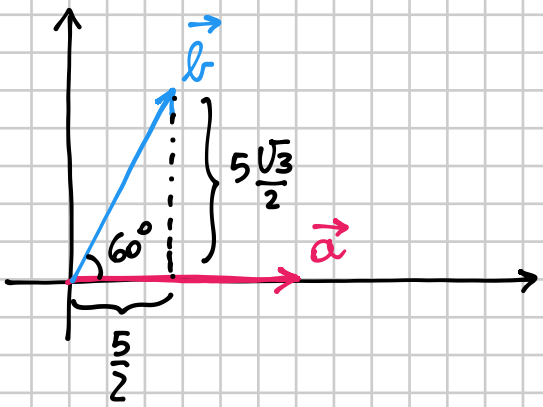
ESEMPIO



$$\vec{a} = (3, 2) \quad \vec{b} = (4, 6)$$

$$\vec{a} \cdot \vec{b} = 3 \cdot 4 + 2 \cdot 6 = 12 + 12 = 24$$

ALTRO ESEMPIO



$$\vec{a} = (6, 0) \qquad |\vec{a}| = 6$$

$$\vec{b} = \left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right) \qquad |\vec{b}| = 5$$

1° MODO = $\vec{a} \cdot \vec{b} = ab \cdot \cos 60° =$

$$= 6 \cdot 5 \cdot \frac{1}{2} = 15$$

2° MODO = $\vec{a} \cdot \vec{b} = (6, 0) \cdot \left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right) =$

$$= 6 \cdot \frac{5}{2} + 0 \cdot \frac{5\sqrt{3}}{2} = 15$$