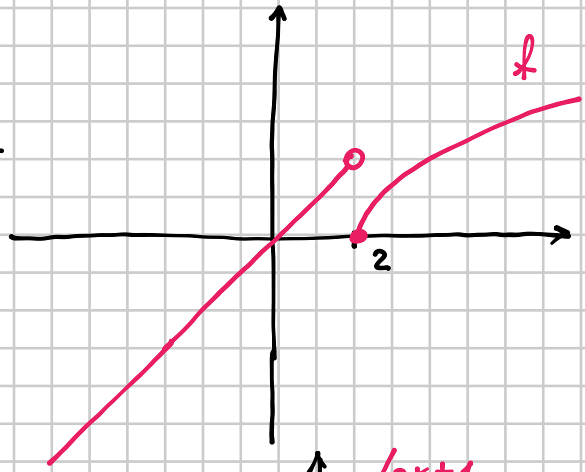
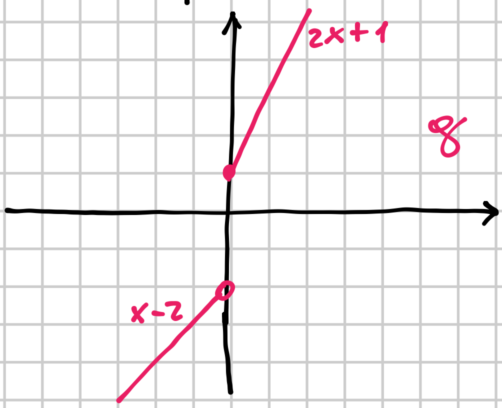
9/11/2021

302 Trova $g \circ f$, con $f(x) = \begin{cases} x & \text{se } x < 2 \\ \sqrt{x-2} & \text{se } x \geq 2 \end{cases}$, $g(x) = \begin{cases} x-2 & \text{se } x < 0 \\ 2x+1 & \text{se } x \geq 0 \end{cases}$. $\left[g \circ f: \begin{cases} 2\sqrt{x-2}+1 & \text{se } x \geq 2 \\ 2x+1 & \text{se } 0 \leq x < 2 \\ x-2 & \text{se } x < 0 \end{cases} \right]$

$f: \mathbb{R} \to \mathbb{R} \qquad f(x) = \begin{cases} x & \text{se} \quad x < 2 \\ \sqrt{x-2} & \text{se} \quad x \geq 2 \end{cases}$



$g: \mathbb{R} \to \mathbb{R} \qquad g(x) = \begin{cases} x-2 & \text{se} \quad x < 0 \\ 2x+1 & \text{se} \quad x \geq 0 \end{cases}$



$x < 0$

$g \circ f: \mathbb{R} \to \mathbb{R} \qquad g(f(x)) = \begin{cases} f(x) - 2 & \text{se} \quad f(x) < 0 \\ 2f(x) + 1 & \text{se} \quad f(x) \geq 0 \end{cases} = \begin{cases} f(x) - 2 & \text{se} \quad x < 0 \\ 2f(x) + 1 & \text{se} \quad x \geq 0 \end{cases}$

$x \geq 0$

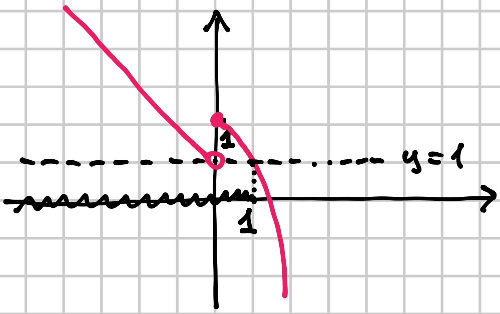
$= \begin{cases} x - 2 & \text{se} \quad x < 0 \\ 2x + 1 & \text{se} \quad 0 \leq x < 2 \\ 2\sqrt{x-2} + 1 & \text{se} \quad x \geq 2 \end{cases}$

Comporre le seguenti funzioni: $g \circ f$

$$f(x) = \begin{cases} -x+1 & \text{se } x<0 \\ -x^2+2 & \text{se } x \geq 0 \end{cases} \qquad g(x) = \begin{cases} x^3-1 & \text{se } x \geq 1 \\ 2x & \text{se } x<1 \end{cases}$$

$$g(f(x)) = \begin{cases} f^3(x) - 1 & \text{se } f(x) \geq 1 \\ 2f(x) & \text{se } f(x) < 1 \end{cases}$$

vanno risolte!



$f(x) \geq 1$ se $x \leq 1$

$f(x) < 1$ se $x > 1$

$$g(f(x)) = \begin{cases} f^3(x) - 1 & \text{se } x \leq 1 \\ 2f(x) & \text{se } x > 1 \end{cases} = \begin{cases} (-x+1)^3 - 1 & \text{se } x<0 \\ (-x^2+2)^3 - 1 & \text{se } 0 \leq x \leq 1 \\ 2(-x^2+2) & \text{se } x > 1 \end{cases}$$

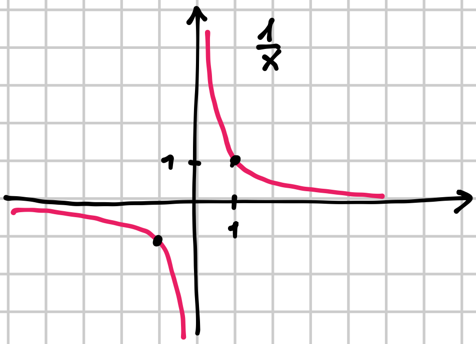
Dimostrare che la funzione $f : \mathbb{R} \setminus \{2\} \to \mathbb{R}$ definita da

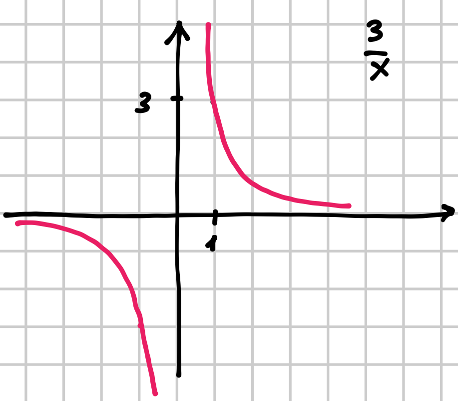
$$f(x) = \frac{x+1}{x-2}$$

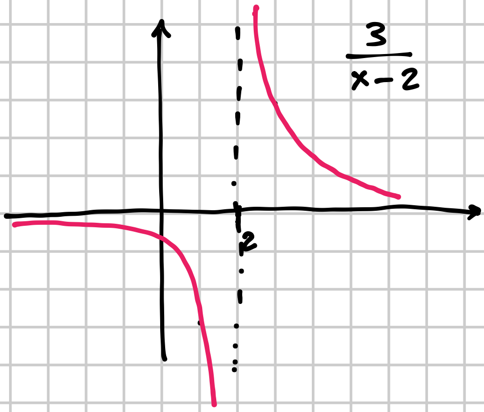
è strettamente decrescente negli intervalli $(-\infty, 2)$ e $(2, +\infty)$.

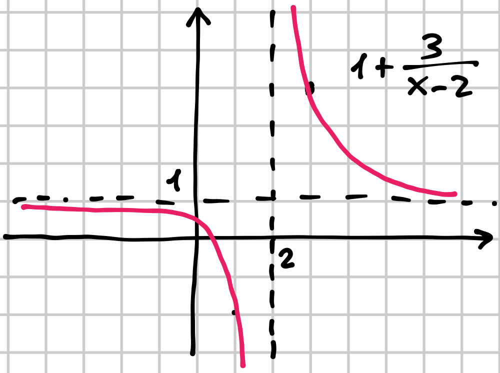
Osserviamo che $f(x) = \dfrac{x+1}{x-2} = \dfrac{x-2+2+1}{x-2} = \dfrac{x-2+3}{x-2} =$

$$= \frac{x-2}{x-2} + \frac{3}{x-2} = 1 + \frac{3}{x-2}$$









1) Prendo l'intervallo $(2, +\infty)$

$$x_1 < x_2 \implies f(x_1) > f(x_2) \quad \forall x_1, x_2 \in (2, +\infty)$$

$$x_1 < x_2 \iff x_1 - 2 < x_2 - 2$$

$$\iff \frac{1}{x_1 - 2} > \frac{1}{x_2 - 2} \iff \frac{3}{x_1 - 2} > \frac{3}{x_2 - 2}$$

$$\iff \underbrace{1 + \frac{3}{x_1 - 2}}_{f(x_1)} > \underbrace{1 + \frac{3}{x_2 - 2}}_{f(x_2)} \iff f(x_1) < f(x_2)$$

quindi f è decrescente strettamente in $(2, +\infty)$