26/11/2021

615 Studia il fascio di rette di equazione $(2k+1)x+(3+k)y+1-2k=0$, determinando le equazioni delle generatrici e le coordinate del centro C. Calcola il valore di k corrispondente alla retta:

a. parallela alla retta di equazione $x+y-1=0$;

b. passante per $P(5;1)$;

c. passante per Q, essendo Q un punto del primo quadrante, vertice del triangolo isoscele PCQ di base PC e area $\frac{441}{40}$.

[generatrici: $x+3y+1=0$, $2x+y-2=0$; $C\left(\frac{7}{5};-\frac{4}{5}\right)$; a) 2; b) -1; c) $-\frac{67}{18}$]

$$(2K+1)x+(3+K)y+1-2K=0$$

$$2Kx+x+3y+Ky+1-2K=0$$

$$x+3y+1+K(2x+y-2)=0$$

1° gen.

2° gen. (escluso)

$$\begin{cases} x+3y+1=0 \\ 2x+y-2=0 \end{cases} \quad \begin{cases} x+3(2-2x)+1=0 \\ y=2-2x \end{cases} \quad \begin{cases} x+6-6x+1=0 \\ y=2-2x \end{cases}$$

$$\begin{cases} -5x=-7 \\ y=2-2x \end{cases} \quad \begin{cases} x=\frac{7}{5} \\ y=2-\frac{14}{5}=-\frac{4}{5} \end{cases} \quad C\left(\frac{7}{5},-\frac{4}{5}\right)$$

a) $/\!/ \; x+y-1=0$ $\qquad (2K+1)x+(3+K)y+1-2K=0$

$m=-\frac{a}{b}=-1$ $\qquad -\frac{2K+1}{3+K}=-1$ $\qquad (K\neq-3)$

$$2K+1=3+K$$

$$\boxed{K=2}$$

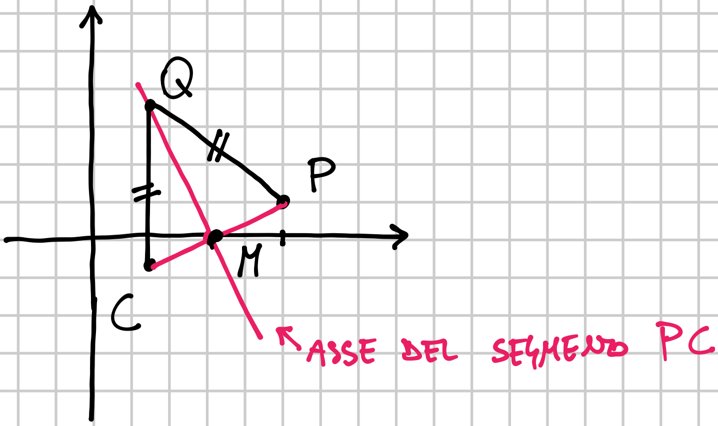
b) passante per $P(5,1)$ $\quad (2k+1)x + (3+k)y + 1 - 2k = 0$

$$(2k+1)\cdot 5 + (3+k)\cdot 1 + 1 - 2k = 0$$

$$10k + 5 + 3 + k + 1 - 2k = 0$$

$$9k = -9 \qquad \boxed{k = -1}$$

c) $Q \in$ I QUADR. $\quad PCQ$ ISOSCELE $\quad$ BASE PC $\quad \mathcal{A} = \dfrac{441}{40}$



Devo trovare l'asse del segmento PC: retta perpendicolare a PC passante per il suo punto medio

$$m_{PC} = \frac{-\frac{4}{5} - 1}{\frac{7}{5} - 5} = \frac{\frac{-4-5}{5}}{\frac{7-25}{5}} = \frac{-9}{-18} = \frac{1}{2}$$

$$m_{\text{ASSE DI } PC} = -2$$

$P(5,1) \quad C\left(\frac{7}{5}, -\frac{4}{5}\right)$

$$M\left(\frac{5+\frac{7}{5}}{2}, \frac{1-\frac{4}{5}}{2}\right) =$$

$$= \left(\frac{32}{10}, \frac{1}{10}\right) = \left(\frac{16}{5}, \frac{1}{10}\right)$$

retta per M di coeff. ang. -2 $\Rightarrow$ $\quad y - \frac{1}{10} = -2\left(x - \frac{16}{5}\right)$ $\quad$ retta che contiene Q

$$y - \frac{1}{10} = -2x + \frac{32}{5}$$

$$y = -2x + \frac{32}{5} + \frac{1}{10}$$

$$y = -2x + \frac{\cancel{65}^{\,13}}{\cancel{10}_{\,2}} \qquad y = -2x + \frac{13}{2}$$

Q deve quindi appartenere alla retta $y = -2x + \frac{13}{2}$, cioè

$$Q\left(\underset{\substack{\uparrow \\ \text{DA TROVARE}}}{x_Q}, -2x_Q + \frac{13}{2}\right)$$

$$\overline{PC} = \sqrt{\left(5-\frac{7}{5}\right)^2 + \left(1+\frac{4}{5}\right)^2} = \sqrt{\left(\frac{18}{5}\right)^2 + \left(\frac{9}{5}\right)^2} =$$

$$= \sqrt{\left(\frac{9}{5}\right)^2\left[2^2+1\right]} = \frac{9}{5}\sqrt{5} \quad \text{BASE DEL TRIANGOLO}$$

$$\text{ALTEZZA TRIANGOLO } (= \overline{QM}) = \frac{2\mathcal{A}}{\overline{PC}} = \frac{2\cdot\frac{441}{40}}{\frac{9}{5}\sqrt{5}} = \frac{\cancel{2}^{\,1}\cdot\cancel{441}^{\,49}}{\cancel{40}_{\cancel{8}\,4}} \cdot \frac{\cancel{5}^{\,1}}{\cancel{9}\sqrt{5}} =$$

$$= \frac{49}{4\sqrt{5}}$$

Impongo che la distanza $\overline{QM}$ sia uguale a $\frac{49}{4\sqrt{5}}$

$$\overline{QM} = \frac{49}{4\sqrt{5}} \iff \overline{QM}^2 = \frac{49^2}{80}$$

$$Q\left(\underset{\substack{\uparrow \\ \text{lo chiamo } x \text{ per semplicità}}}{x_Q}, -2x_Q + \frac{13}{2}\right) \qquad M\left(\frac{16}{5}, \frac{1}{10}\right)$$

$$\left(x - \frac{16}{5}\right)^2 + \left(-2x + \frac{13}{2} - \frac{1}{10}\right)^2 = \frac{\overset{49^2}{2401}}{80}$$

$x > 0$ perché nel I QUAD.

$$\left(x-\frac{16}{5}\right)^2+\left(-2x+\frac{32}{5}\right)^2=\frac{2401}{80}$$

$$x^2+\frac{256}{25}-\frac{32}{5}x+4x^2+\frac{1024}{25}-\frac{128}{5}x-\frac{2401}{80}=0$$

$$\frac{1280}{25}=\frac{256}{5}$$

$$5x^2-\frac{160}{5}x+\frac{4096-2401}{80}=0$$

$$5x^2-32x+\frac{\overset{339}{\cancel{1695}}}{\underset{16}{\cancel{80}}}=0 \qquad 5x^2-32x+\frac{339}{16}=0$$

$$\frac{\Delta}{4}=256-\frac{1695}{16}=\frac{2401}{16}=\left(\frac{49}{4}\right)^2$$

$$x=\frac{16\pm\frac{49}{4}}{5}= \begin{cases} \dfrac{\frac{64-49}{4}}{5}=\dfrac{15}{20}=\dfrac{3}{4} \\[2ex] \dfrac{\frac{64+49}{4}}{5}=\dfrac{113}{20} \end{cases}$$

entrambi positivi

$$x=\frac{3}{4} \Rightarrow y=-2x+\frac{13}{2}=-\cancel{2}^{1}\cdot\frac{3}{\cancel{4}_2}+\frac{13}{2}=\frac{10}{2}=5$$

$$x=\frac{113}{20} \Rightarrow y=-2x+\frac{13}{2}=-\cancel{2}\cdot\frac{113}{\cancel{20}_{10}}+\frac{13}{2}=\frac{-113+65}{10}=-\frac{48}{10}<0$$

NON ACCETTABILE

perché $Q\in$ I QUADR.

Quindi $Q\left(\frac{3}{4},5\right)$

Per trovare K sostituisco Q nell'eq. del fascio:

$$(2K+1)x+(3+K)y+1-2K=0 \Rightarrow (2K+1)\cdot\frac{3}{4}+(3+K)\cdot 5+1-2K=0$$

$$(2K+1)\cdot\frac{3}{4}+(3+K)\cdot 5+1-2K=0$$

$$\frac{3}{2}K+\frac{3}{4}+15+5K+1-2K=0$$

$$\frac{9}{2}K=-\frac{67}{4} \Rightarrow \boxed{K=-\frac{67}{18}}$$