264 Scrivi le equazioni delle rette t_1 e t_2 tangenti alla parabola di equazione $x = \frac{1}{2}y^2 - 2y$ e passanti per $P(-2;3)$. Condotta poi la tangente t_3 nel punto della parabola di ordinata 1, trova l'area del triangolo definito da t_1, t_2, t_3.

$\left[x=-2;\ x-2y+8=0;\ 2x+2y+1=0;\ \text{area}=\frac{3}{4}\right]$

$$x = \frac{1}{2}y^2 - 2y \qquad P(-2,3) \qquad y-3 = m(x+2)$$

$$\begin{cases} x = \frac{1}{2}y^2 - 2y \\ y-3 = mx + 2m \end{cases} \qquad \begin{cases} x = \frac{1}{2}y^2 - 2y \\ mx = y - 3 - 2m \end{cases} \qquad \begin{cases} x = \frac{1}{2}y^2 - 2y \\ x = \frac{y-3}{m} - 2 \end{cases}$$

$m \neq 0$

$$\frac{y-3}{m} - 2 = \frac{1}{2}y^2 - 2y$$

$$\frac{1}{2}y^2 - 2y - \frac{1}{m}y + \frac{3}{m} + 2 = 0$$

$$\frac{1}{2}y^2 - \left(2 + \frac{1}{m}\right)y + \frac{3}{m} + 2 = 0$$

pongo $\Delta = 0$

$$\left(2 + \frac{1}{m}\right)^2 - \cancel{4}^{2} \cdot \frac{1}{\cancel{2}}\left(\frac{3}{m} + 2\right) = 0$$

$$\cancel{4} + \frac{1}{m^2} + \frac{4}{m} - \frac{6}{m} - \cancel{4} = 0$$

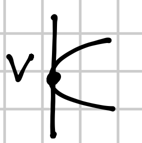
$$\frac{1 + 4m - 6m}{\cancel{m^2}} = 0$$

$$-2m + 1 = 0$$

$$m = \frac{1}{2}$$

SICCOME È DI 1° GRADO, SIGNIFICA CHE L'ALTRA TANGENTE È VERTICALE

$m = \infty$



$x = x_V$

$\boxed{x = -2}$

$$y - 3 = m(x+2) \qquad y - 3 = \frac{1}{2}(x+2)$$

$$y = \frac{1}{2}x + 1 + 3 \qquad y = \frac{1}{2}x + 4$$

$\hookrightarrow 2y = x + 8$ $\quad \boxed{x - 2y + 8 = 0}$

$$x = \frac{1}{2}y^2 - 2y \qquad T(?, 1) \Rightarrow T\left(-\frac{3}{2}, 1\right)$$

$$x = \frac{1}{2}\cdot 1^2 - 2\cdot 1 = \frac{1}{2} - 2 = -\frac{3}{2}$$

$$\begin{cases} y - 1 = m\left(x + \frac{3}{2}\right) \\ x = \frac{1}{2}y^2 - 2y \end{cases}$$

$$y - 1 = m\left(\frac{1}{2}y^2 - 2y + \frac{3}{2}\right)$$

$$y - 1 = \frac{1}{2}my^2 - 2my + \frac{3}{2}m$$

$$\frac{1}{2}my^2 - 2my - y + \frac{3}{2}m + 1 = 0$$

$$\frac{1}{2}my^2 - (2m+1)y + \frac{3}{2}m + 1 = 0$$

$\Delta = 0$ $$(2m+1)^2 - 4\cdot\frac{1}{2}m\left(\frac{3}{2}m + 1\right) = 0$$

$$4m^2 + 1 + 4m - 3m^2 - 2m = 0$$

$$m^2 + 2m + 1 = 0 \qquad (m+1)^2 = 0 \Rightarrow m = -1$$

tangente t_3 $\quad y - 1 = -x - \frac{3}{2} \qquad y = -x - \frac{1}{2}$

t_1: $x = -2$ $\quad \begin{cases} x = -2 \\ y = \frac{1}{2}x + 4 \end{cases} \quad \begin{cases} x = -2 \\ y = 3 \end{cases} \quad P(-2, 3)$

t_2: $y = \frac{1}{2}x + 4$

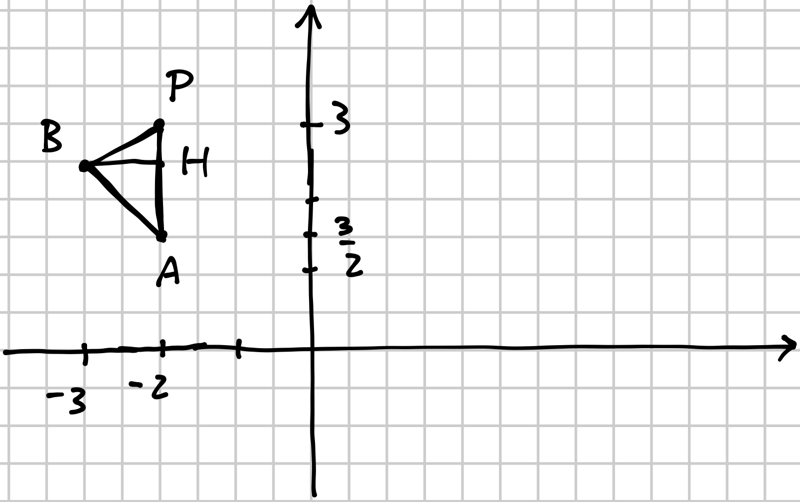
t_3: $y = -x - \frac{1}{2}$ $\quad \begin{cases} x = -2 \\ y = -x - \frac{1}{2} \end{cases} \quad \begin{cases} x = -2 \\ y = \frac{3}{2} \end{cases} \quad A\left(-2, \frac{3}{2}\right)$

$\begin{cases} y = \frac{1}{2}x + 4 \\ y = -x - \frac{1}{2} \end{cases} \quad \frac{1}{2}x + 4 = -x - \frac{1}{2} \quad \frac{3}{2}x = -\frac{9}{2} \quad x = -3 \quad y = \frac{5}{2} \quad B\left(-3, \frac{5}{2}\right)$

$P(-2, 3)$

$A\left(-2, \frac{3}{2}\right)$

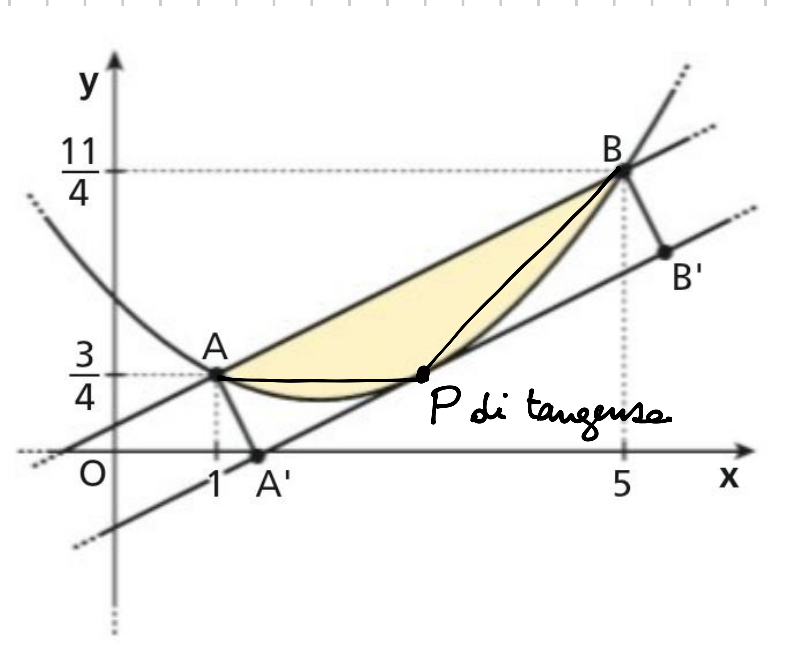
$B\left(-3, \frac{5}{2}\right)$



$\overline{BH} = 1 \qquad \overline{AP} = \left|3 - \frac{3}{2}\right| = \frac{3}{2}$

$$\mathcal{A} = \frac{1 \cdot \frac{3}{2}}{2} = \frac{3}{4}$$

AREA DEL SEGMENTO PARABOLICO

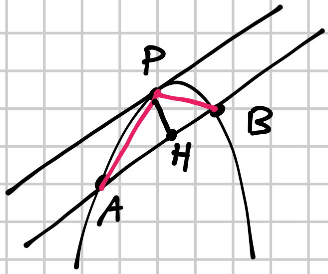


$$\mathcal{A}_{\substack{\text{SEGMENTO APB}\\ \text{PARABOLICO}}} = \frac{4}{3} \underbrace{\mathcal{A}_{APB}}_{\text{TRIANGOLO}}$$

TEOREMA

DOVUTO AD ARCHIMEDE

273 Trova l'area del segmento parabolico definito dalla parabola di equazione $y = -\frac{1}{2}x^2 - 2x - 3$ e dalla retta che congiunge i due punti della parabola di ascissa -7 e -1. [18]



Trovo i punti A e B

$$A \begin{cases} y = -\frac{1}{2}\cdot 49 + 14 - 3 = -\frac{49}{2} + 11 = -\frac{27}{2} \\ x = -7 \end{cases} \qquad A\left(-7, -\frac{27}{2}\right)$$

$$B\left(-1, -\frac{3}{2}\right)$$

$$B \begin{cases} x = -1 \\ y = -\frac{1}{2} + 2 - 3 = -\frac{3}{2} \end{cases}$$

Trovo l'equazione della retta AB

$$\frac{y + \frac{27}{2}}{-\frac{3}{2} + \frac{27}{2}} = \frac{x+7}{-1+7} \qquad \frac{y + \frac{27}{2}}{\cancel{12}_{\,2}} = \frac{x+7}{\cancel{6}_{\,1}}$$

$$y + \frac{27}{2} = 2x + 14$$

$$y = 2x + 14 - \frac{27}{2}$$

$$y = 2x + \frac{1}{2} \qquad (4x - 2y + 1 = 0)$$

$$\begin{cases} y = 2x + K \\ y = -\frac{1}{2}x^2 - 2x - 3 \end{cases}$$

$$2x + K = -\frac{1}{2}x^2 - 2x - 3 \qquad \frac{1}{2}x^2 + 4x + K + 3 = 0$$

$$\frac{\Delta}{4} = 0 \Rightarrow \quad 4 - \frac{1}{2}(K+3) = 0 \qquad 4 - \frac{1}{2}K - \frac{3}{2} = 0 \qquad -\frac{1}{2}K = \frac{3}{2} - 4$$

$$-\frac{1}{2}K = -\frac{5}{2} \Rightarrow K = 5 \qquad y = 2x + 5 \text{ tangente}$$

Trovo il punto di tangenza P

$$\begin{cases} y = -\frac{1}{2}x^2 - 2x - 3 \\ y = 2x + 5 \end{cases}$$

$$2x + 5 = -\frac{1}{2}x^2 - 2x - 3$$

$$\frac{1}{2}x^2 + 4x + 8 = 0 \qquad \frac{\Delta}{4} = 4 - 4 = 0$$

$$x = \frac{-2 \pm \sqrt{0}}{\frac{1}{2}} = -4$$

$$\begin{cases} x = -4 \\ y = 2(-4) + 5 = -3 \end{cases} \qquad P(-4, -3)$$

Trovo $\overline{PH}$, distanza di P dalla retta AB $\quad 4x - 2y + 1 = 0$

$$\overline{PH} = \frac{|4(-4) - 2(-3) + 1|}{\sqrt{4^2 + (-2)^2}} = \frac{|-16 + 6 + 1|}{\sqrt{16 + 4}} = \frac{9}{\sqrt{20}} = \frac{9}{2\sqrt{5}}$$

Trovo la distanza $\overline{AB}$ $\quad A\left(-7, -\frac{27}{2}\right) \quad B\left(-1, -\frac{3}{2}\right)$

$$\overline{AB} = \sqrt{(-7+1)^2 + \left(-\frac{27}{2} + \frac{3}{2}\right)^2} = \sqrt{36 + 144} = \sqrt{180} = 6\sqrt{5}$$

$$\mathcal{A}_{\substack{\text{SEGMENTO PAR.} \\ ABP}} = \frac{4}{3}\mathcal{A}_{ABP} = \frac{4}{3} \cdot \frac{1}{2}\overline{AB} \cdot \overline{PH} = \frac{\cancel{4}}{\cancel{3}} \cdot \frac{1}{\cancel{2}} \cdot 6\cancel{\sqrt{5}} \cdot \frac{\cancel{9}^{\,3}}{\cancel{2}\cancel{\sqrt{5}}} = \boxed{18}$$