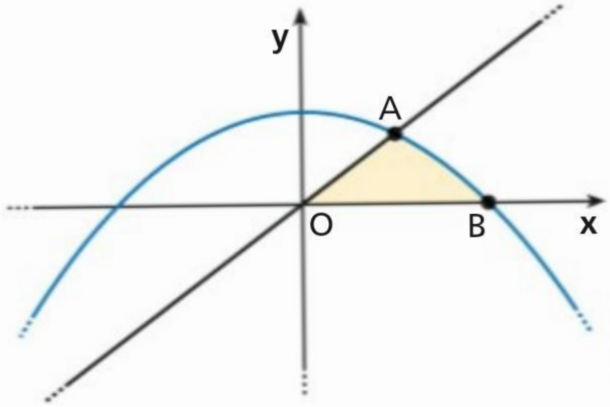
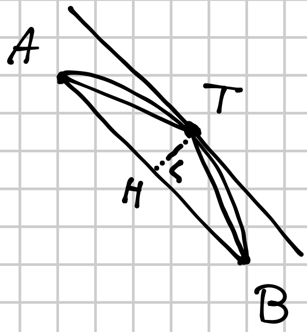
279 Calcola l'area del triangolo mistilineo OAB rappresentato nella figura, sapendo che la parabola ha equazione $y = -\frac{1}{8}x^2 + 2$ e la retta $y = \frac{3}{4}x$.



$\left[\frac{19}{6}\right]$



$$A \begin{cases} y = -\frac{1}{8}x^2 + 2 \\ y = \frac{3}{4}x \end{cases} \qquad \frac{3}{4}x = -\frac{1}{8}x^2 + 2 \qquad \frac{1}{8}x^2 + \frac{3}{4}x - 2 = 0$$

$$x^2 + 6x - 16 = 0 \qquad \frac{\Delta}{4} = 9 + 16 = 25$$

$$A\left(2, \frac{3}{2}\right) \qquad x = -3 \pm 5 = \begin{cases} -8 \\ 2 \end{cases}$$

$$B \begin{cases} y = -\frac{1}{8}x^2 + 2 \\ y = 0 \end{cases} \qquad -\frac{1}{8}x^2 + 2 = 0 \qquad -\frac{1}{8}x^2 = -2 \qquad x = 4 \qquad B(4, 0)$$

retta AB $\qquad \dfrac{y - \frac{3}{2}}{0 - \frac{3}{2}} = \dfrac{x - 2}{4 - 2} \qquad 2y - 3 = -\frac{3}{2}(x - 2)$

$$2y - 3 = -\frac{3}{2}x + 3$$

$$3x + 4y - 12 = 0 \qquad m = -\frac{3}{4}$$

Trovo la parallela ad AB tangente alla parabola

$$\begin{cases} y = -\frac{3}{4}x + K \\ y = -\frac{1}{8}x^2 + 2 \end{cases} \qquad -\frac{1}{8}x^2 + 2 = -\frac{3}{4}x + K$$

$$x^2 - 16 = +6x - 8K$$

$$x^2 - 6x + 8K - 16 = 0$$

$$\frac{\Delta}{4} = 0 \qquad 9 - (8K - 16) = 0 \qquad 9 - 8K + 16 = 0 \qquad K = \frac{25}{8}$$

$k = \frac{25}{8}$ nell'eq. risolvente

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$ $\quad x = 3$

prima coordinata del punto di tangenza

$y = -\frac{1}{8}x^2 + 2 =$

$= -\frac{9}{8} + 2 = \frac{7}{8}$

$T\left(3, \frac{7}{8}\right)$

$3x + 4y - 12 = 0$ retta AB

$\overline{TH}$ = distanza di T dalla retta AB $= \dfrac{\left|3 \cdot 3 + 4 \cdot \frac{7}{8} - 12\right|}{\sqrt{3^2 + 4^2}} = \dfrac{\left|9 + \frac{7}{2} - 12\right|}{5} = \dfrac{\frac{1}{2}}{5} =$

$= \frac{1}{10}$

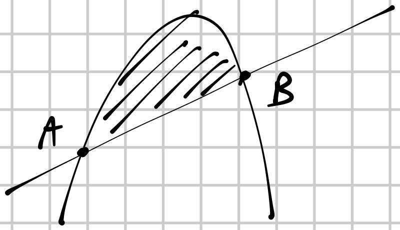
$\overline{AB} = \sqrt{(2-4)^2 + \left(\frac{3}{2} - 0\right)^2} = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

$\mathcal{A}_{\substack{\text{SEGM. PAR.} \\ AB}} = \frac{4}{3} \mathcal{A}_{ABT} = \frac{\cancel{4}}{3} \cdot \frac{1}{\cancel{2}} \cdot \frac{5}{\cancel{2}} \cdot \frac{1}{\cancel{10}_{2}} = \frac{1}{6}$

$\mathcal{A}_{OAB} = \frac{1}{2} \cdot 4 \cdot \frac{3}{2} = 3$

$\mathcal{A}_{TOT} = 3 + \frac{1}{6} = \boxed{\frac{19}{6}}$

275 Calcola l'area della parte di piano delimitata dalla parabola di equazione $y = -\frac{1}{3}(x+1)(x-3)$ e dalla retta di equazione $y = \frac{1}{3}(x+1)$. $\left[\frac{3}{2}\right]$



$$y = -\frac{1}{3}(x^2 - 3x + x - 3) =$$

$$= -\frac{1}{3}x^2 + \frac{2}{3}x + 1$$

Trovo A e B

$$\begin{cases} y = -\frac{1}{3}x^2 + \frac{2}{3}x + 1 \\ y = \frac{1}{3}x + \frac{1}{3} \end{cases}$$

$$\frac{1}{3}x + \frac{1}{3} = -\frac{1}{3}x^2 + \frac{2}{3}x + 1$$

$$\frac{1}{3}x^2 - \frac{1}{3}x - \frac{2}{3} = 0$$

$$x^2 - x - 2 = 0 \qquad (x-2)(x+1) = 0$$

$$\begin{cases} x = -1 \\ y = 0 \end{cases} \vee \begin{cases} x = 2 \\ y = 1 \end{cases}$$

$A(-1, 0) \quad B(2, 1)$

$$\begin{cases} y = \frac{1}{3}x + k \\ y = -\frac{1}{3}x^2 + \frac{2}{3}x + 1 \end{cases}$$

$$\frac{1}{3}x + k = -\frac{1}{3}x^2 + \frac{2}{3}x + 1$$

$$\frac{1}{3}x^2 - \frac{1}{3}x + k - 1 = 0$$

$$\Delta = 0 \Rightarrow \left(-\frac{1}{3}\right)^2 - 4 \cdot \frac{1}{3}(k-1) = 0$$

$$\frac{1}{9} - \frac{4}{3}k + \frac{4}{3} = 0 \qquad -\frac{4}{3}k = -\frac{1}{9} - \frac{4}{3}$$

$$-\frac{12k}{\cancel{9}} = \frac{-1-12}{\cancel{9}} \qquad k = \frac{13}{12}$$

$$\frac{1}{3}x^2-\frac{1}{3}x+\frac{13}{12}-1=0$$

$$\frac{1}{3}x^2-\frac{1}{3}x+\frac{1}{12}=0$$

$$4x^2-4x+1=0 \qquad (2x-1)^2=0 \qquad \begin{cases} x=\frac{1}{2} \\ y=-\frac{1}{3}x^2+\frac{2}{3}x+1= \end{cases}$$

$$=-\frac{1}{3}\cdot\frac{1}{4}+\frac{2}{3}\cdot\frac{1}{2}+1=$$

$$=-\frac{1}{12}+\frac{1}{3}+1=$$

$$T\left(\frac{1}{2},\frac{15}{12}\right) \quad A(-1,0) \quad B(2,1)$$

$$=\frac{-1+4+12}{12}=\frac{15}{12}$$

$$\mathcal{A}_{ATB}=\frac{1}{2}\begin{vmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ \frac{1}{2} & \frac{15}{12} & 1 \end{vmatrix}=\frac{1}{2}\left|\begin{array}{ccc:cc} -1 & 0 & 1 & -1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ \frac{1}{2} & \frac{15}{12} & 1 & \frac{1}{2} & \frac{15}{12} \end{array}\right|=$$

$$=\frac{1}{2}\left(-1+0+\frac{15}{6}-\frac{1}{2}+\frac{15}{12}-0\right)=\frac{1}{2}\left(\frac{-12+30-6+15}{12}\right)=$$

$$=\frac{1}{2}\cdot\frac{27}{12}=\frac{27}{24}$$

$$\mathcal{A}_{SEGMENTO}=\frac{\cancel{4}}{\cancel{3}}\cdot\frac{\cancel{27}^{\,\cancel{9}\,3}}{\cancel{24}_{\,\cancel{6}\,2}}=\boxed{\frac{3}{2}}$$