7/3/2022

295 Trova le equazioni delle circonferenze passanti per i punti $A(1;-4)$ e $B(3;0)$ e tangenti alla retta di equazione $2x+y+3=0$.

$[x^2+y^2-4x+4y+3=0;$
$4x^2+4y^2-46x+31y+102=0]$

$$x^2+y^2+ax+by+c=0$$

$A(1,-4)$
$B(3,0)$

$$\begin{cases} 1+16+a-4b+c=0 \\ 9+3a+c=0 \end{cases} \quad \begin{cases} 17+a-4b-9-3a=0 \\ c=-9-3a \end{cases}$$

$$\begin{cases} -2a-4b+8=0 \\ c=-3a-9 \end{cases} \quad \begin{cases} 4b=8-2a \\ c=-3a-9 \end{cases} \quad \begin{cases} b=2-\frac{1}{2}a \\ c=-3a-9 \end{cases}$$

$$x^2+y^2+ax+\left(2-\frac{1}{2}a\right)y+(-3a-9)=0$$

tangente $2x+y+3=0$

$$\begin{cases} x^2+y^2+ax+\left(2-\frac{1}{2}a\right)y+(-3a-9)=0 \\ y=-2x-3 \end{cases}$$

$$x^2+(-2x-3)^2+ax+\left(2-\frac{1}{2}a\right)(-2x-3)-3a-9=0$$

$$x^2+4x^2+\cancel{9}+\underset{\sim}{12x}+\underset{\sim}{ax}-\underset{\sim}{4x}-6+\underset{\sim}{ax}+\frac{3}{2}a-3a-\cancel{9}=0$$

$$5x^2+(8+2a)x-\frac{3}{2}a-6=0$$

$$5x^2+2(a+4)x-\frac{3}{2}a-6=0 \qquad \text{pongo } \frac{\Delta}{4}=0$$

$$5x^2+2(a+4)x-\frac{3}{2}a-6=0 \qquad \text{pongo } \frac{\Delta}{4}=0$$

$$(a+4)^2-5\left(-\frac{3}{2}a-6\right)=0$$

$$a^2+16+8a+\frac{15}{2}a+30=0$$

$$a^2+\frac{31}{2}a+46=0$$

$$2a^2+31a+92=0$$

$$\Delta=31^2-8\cdot 92=961-736=225=15^2$$

$$a=\frac{-31\pm 15}{4}= \begin{cases} -\frac{46}{4}=-\frac{23}{2} \\ -\frac{16}{4}=-4 \end{cases}$$

$$x^2+y^2+ax+\left(2-\frac{1}{2}a\right)y+(-3a-9)=0$$

$$a=-\frac{23}{2} \Rightarrow x^2+y^2-\frac{23}{2}x+\left(2+\frac{23}{4}\right)y+\frac{69}{2}-9=0$$

$$\boxed{x^2+y^2-\frac{23}{2}x+\frac{31}{4}y+\frac{51}{2}=0}$$

$$a=-4 \Rightarrow \boxed{x^2+y^2-4x+4y+3=0}$$