407 Scrivi l'equazione della circonferenza γ passante per l'origine O e tangente alla retta di equazione

$$-3x + 2y - 13 = 0$$

nel suo punto di ascissa -1.
Detti A e B i punti di intersezione di γ con gli assi cartesiani, determina un punto P sulla semicirconferenza che non contiene l'origine in modo che l'area del quadrilatero $OAPB$ sia uguale a 17.

$\left[x^2 + y^2 - 4x - 6y = 0;\ P_1(5;1),\ P_2\left(\frac{17}{13};\ \frac{85}{13}\right)\right]$

Punto di tangenza $T(-1, 5)$

$$-3(-1) + 2y - 13 = 0$$

$$2y = 10 \Rightarrow y = 5$$

$$x^2 + y^2 + ax + by + c = 0$$

$O(0,0) \Rightarrow$
$T(-1,5)$

$$\begin{cases} c = 0 \\ 1 + 25 - a + 5b + c = 0 \end{cases} \qquad \begin{cases} c = 0 \\ a = 26 + 5b \end{cases}$$

$$\begin{cases} x^2 + y^2 + (26 + 5b)x + by = 0 \\ -3x + 2y - 13 = 0 \Rightarrow y = \frac{3}{2}x + \frac{13}{2} \end{cases}$$

$$x^2 + \left(\frac{3}{2}x + \frac{13}{2}\right)^2 + (26 + 5b)x + b\left(\frac{3}{2}x + \frac{13}{2}\right) = 0$$

$$x^2 + \frac{9}{4}x^2 + \frac{169}{4} + \frac{39}{2}x + \left(26 + 5b + \frac{3}{2}b\right)x + \frac{13}{2}b = 0$$

$$\frac{13}{4}x^2 + \left(\frac{39}{2} + 26 + \frac{13}{2}b\right)x + \frac{13}{2}b + \frac{169}{4} = 0$$

$$\frac{1}{4}x^2 + \left(\frac{3}{2} + 2 + \frac{1}{2}b\right)x + \frac{1}{2}b + \frac{13}{4} = 0$$

$$\frac{1}{4}x^2 + \frac{7 + b}{2}x + \frac{1}{2}b + \frac{13}{4} = 0$$

$$x^2 + 2(b + 7)x + 2b + 13 = 0 \qquad \frac{\Delta}{4} = 0 \qquad (b + 7)^2 - 2b - 13 = 0$$

$$b^2 + 49 + 14b - 2b - 13 = 0$$

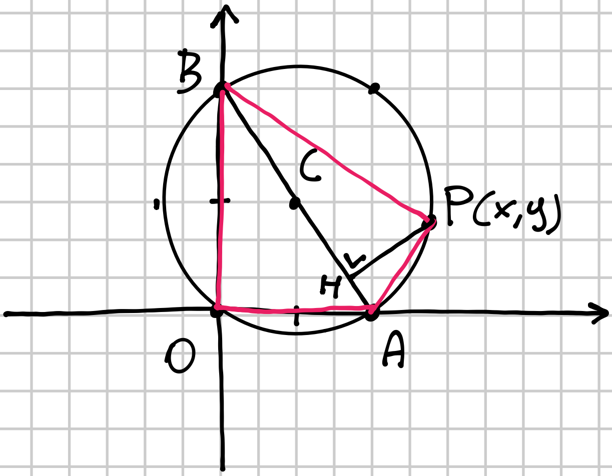
$$b^2+49+14b-2b-13=0$$

$$b^2+12b+36=0$$

$$(b+6)^2=0 \qquad b=-6$$

$$x^2+y^2+(26+5b)x+by=0 \Rightarrow \boxed{x^2+y^2-4x-6y=0}$$

$$C(2,3)$$



$$\mathcal{A}_{OAPB} = \underbrace{\mathcal{A}_{OAB}}_{12} + \mathcal{A}_{APB} = 17$$

$$\begin{cases} x^2+y^2-4x-6y=0 \\ y=0 \end{cases} \quad \begin{cases} x^2-4x=0 \\ y=0 \end{cases} \quad \begin{cases} x(x-4)=0 \\ y=0 \end{cases} \quad \begin{cases} x=0 \lor x=4 \\ y=0 \end{cases}$$

$$A(4,0)$$

$$\begin{cases} x^2+y^2-4x-6y=0 \\ x=0 \end{cases} \quad \begin{cases} y^2-6y=0 \\ x=0 \end{cases} \quad \begin{cases} y(y-6)=0 \\ x=0 \end{cases} \quad \begin{cases} y=0 \lor y=6 \\ x=0 \end{cases}$$

$$B(0,6)$$

retta AB $\quad y=-\frac{3}{2}x+6 \Rightarrow 2y=-3x+12 \qquad 3x+2y-12=0$

$$\overline{PH} = \frac{|3x+2y-12|}{\sqrt{3^2+2^2}} = \frac{|3x+2y-12|}{\sqrt{13}}$$

$$\overline{AB} = \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$

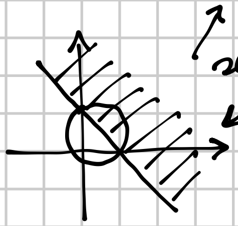
$$\mathcal{A}_{APB} = \frac{1}{2}\,\overline{AB}\cdot\overline{PH} = \frac{1}{\cancel{2}}\,\cancel{2}\cancel{\sqrt{13}}\cdot\frac{|3x+2y-12|}{\cancel{\sqrt{13}}} = |3x+2y-12|$$

$$\mathcal{A}_{OAPB} = 12 + |3x+2y-12| = 17$$

$$\Downarrow$$

$$\begin{cases} |3x+2y-12| = 5 \\ x^2+y^2-4x-6y=0 \\ y > -\frac{3}{2}x+6 \end{cases}$$

semipiano superiore



$$\begin{cases} 3x+2y-12 = \pm 5 \\ x^2+y^2-4x-6y=0 \end{cases}$$

$\boxed{1}$ $\begin{cases} 3x+2y-12=5 \\ x^2+y^2-4x-6y=0 \end{cases}$ $\qquad$ $\begin{cases} y = -\frac{3}{2}x+\frac{17}{2} \\ x^2+\left(-\frac{3}{2}x+\frac{17}{2}\right)^2-4x-6\left(-\frac{3}{2}x+\frac{17}{2}\right)=0 \end{cases}$

$$x^2+\frac{9}{4}x^2+\frac{289}{4}-\frac{51}{2}x-4x+9x-51=0$$

$$\frac{13}{4}x^2-\frac{41}{2}x+\frac{85}{4}=0 \qquad 13x^2-82x+85=0$$

$$\frac{\Delta}{4} = 41^2-13\cdot 85 = 576 = 24^2$$

$$x = \frac{41\pm 24}{13} = \begin{cases} \frac{17}{13} \\ \frac{65}{13} = 5 \end{cases}$$

$\boxed{P_1(5,1)}$ $\qquad$ $\boxed{P_2\left(\frac{17}{13},\frac{85}{13}\right)}$

$-\frac{3}{2}\cdot 5+\frac{17}{2}$ $\qquad$ $-\frac{3}{2}\cdot\frac{17}{13}+\frac{17}{2} = \frac{-51+221}{26} = \frac{170}{26}$

2]

$$\begin{cases} 3x+2y-12=-5 \\ x^2+y^2-4x-6y=0 \end{cases} \qquad \begin{cases} y=-\frac{3}{2}x+\frac{7}{2} \\ x^2+\left(-\frac{3}{2}x+\frac{7}{2}\right)^2-4x-6\left(-\frac{3}{2}x+\frac{7}{2}\right)=0 \end{cases}$$

$$x^2+\frac{9}{4}x^2+\frac{49}{4}-\frac{21}{2}x-4x+9x-21=0$$

$$\frac{13}{4}x^2-\frac{11}{2}x-\frac{35}{4}=0 \qquad 13x^2-22x-35=0$$

$$\frac{\Delta}{4}=121+455=576$$

$$x=\frac{11\pm 24}{13}= \begin{cases} -1 & \text{N.A. perché P deve essere nel I QUADR.} \\ \frac{35}{13} & \Rightarrow y=-\frac{3}{2}\cdot\frac{35}{13}+\frac{7}{2}=\frac{-105+91}{26}=-\frac{14}{26} \text{ N.A.} \end{cases}$$

perché P deve essere nel I QUAD.