8/4/2022

186 $2^{x+2} - 4 \cdot 5^{x+2} = 25 \cdot 5^{x} - 4 \cdot 2^{x}$ [−3]

$$2^{x+2} + 4 \cdot 2^{x} = 25 \cdot 5^{x} + 4 \cdot 5^{x+2}$$

$$2^{x} \cdot 2^{2} + 4 \cdot 2^{x} = 25 \cdot 5^{x} + 4 \cdot 5^{x} \cdot 5^{2}$$

$$2^{x}(2^{2} + 4) = 5^{x}(25 + 4 \cdot 5^{2})$$

$$8 \cdot 2^{x} = 125 \cdot 5^{x}$$

$$\frac{2^{x}}{5^{x}} = \frac{125}{8}$$

$$\left(\frac{2}{5}\right)^{x} = \left(\frac{2}{5}\right)^{-3} \qquad \boxed{x = -3}$$

185 $21 \cdot 3^{x} - 2^{x+3} = 3^{x+1}$ [−2]

$$21 \cdot 3^{x} - 3^{x+1} = 2^{x+3}$$

$$3^{x}(21 - 3) = 2^{3} \cdot 2^{x}$$

$$\frac{3^{x}}{2^{x}} = \frac{2^{3}}{18} \qquad \frac{3^{x}}{2^{x}} = \frac{2^{\cancel{3}\,2}}{\cancel{2} \cdot 9} \qquad \left(\frac{3}{2}\right)^{x} = \left(\frac{3}{2}\right)^{-2}$$

$$\boxed{x = -2}$$

189 $8 + 2^{x+1} = 2^{2x}$

$2^{2x} - 2^{x+1} - 8 = 0$

$(2^x)^2 - 2^x \cdot 2 - 8 = 0$ $\qquad 2^x = t$

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$t = 4 \Rightarrow 2^x = 4 \qquad x = 2$

$t = -2 \Rightarrow 2^x = -2$

IMPOSSIBILE

$\boxed{x = 2}$

192 $5^{2x} - 5^x = 5^{x-2} - \frac{1}{25}$ $\qquad$ [0; – 2]

$5^{2x} - 5^x - 5^{x-2} + \frac{1}{25} = 0$

$(5^x)^2 - 5^x - 5^x \cdot 5^{-2} + \frac{1}{25} = 0$ $\qquad 5^x = t$

$t^2 - t - \frac{1}{25} t + \frac{1}{25} = 0$

$t(t-1) - \frac{1}{25}(t-1) = 0$

$(t-1)\left(t - \frac{1}{25}\right) = 0$

$t = 1 \qquad 5^x = 1 \qquad x = 0$

$t = \frac{1}{25} \qquad 5^x = \frac{1}{25} \qquad 5^x = 5^{-2} \Rightarrow x = -2$

$\boxed{x = 0 \lor x = -2}$

205 $5^{x+2} - 4 \cdot 5^{1-x} - 30 = -5^{2-x}$ $[0; -1]$

$$5^x \cdot 5^2 - 4 \cdot 5 \cdot 5^{-x} - 30 + 5^2 \cdot 5^{-x} = 0$$

$$25 \cdot 5^x - \frac{20}{5^x} - 30 + \frac{25}{5^x} = 0 \qquad 5^x = t$$

$$25t - \frac{20}{t} - 30 + \frac{25}{t} = 0$$

$$5t - \frac{4}{t} - 6 + \frac{5}{t} = 0$$

$$5t^2 - 4 - 6t + 5 = 0$$

$$5t^2 - 6t + 1 = 0$$

$$\frac{\Delta}{4} = 9 - 5 = 4 \qquad t = \frac{3 \pm 2}{5} = \begin{cases} 1 \\ \frac{1}{5} \end{cases}$$

$$5^x = 1 \Rightarrow x = 0$$

$$5^x = \frac{1}{5} \Rightarrow x = -1$$

$$\boxed{x = 0 \quad \vee \quad x = -1}$$

207 $\dfrac{4}{2^x-1}+\dfrac{3}{2^x+1}=5$ [1]

$$\frac{4}{t-1}+\frac{3}{t+1}=5$$

$t=2^x$

C.E.

$t\neq 1 \Rightarrow 2^x\neq 1 \Rightarrow \boxed{x\neq 0}$

$t\neq -1 \Rightarrow 2^x\neq -1 \quad \forall x$

$$\frac{4(t+1)+3(t-1)}{\cancel{(t-1)(t+1)}}=\frac{5(t^2-1)}{\cancel{(t-1)(t+1)}}$$

$$4t+4+3t-3=5t^2-5$$

$$5t^2-7t-6=0$$

$\Delta=49+120=169=13^2$

$$t=\frac{7\pm 13}{10}= \begin{cases} -\dfrac{6}{10} & \text{N.ACC.} \quad 2^x=-\dfrac{6}{10} \text{ IMP.} \\ \dfrac{20}{10}=2 \Rightarrow 2^x=2 \end{cases}$$

$$\Downarrow$$

$$\boxed{x=1}$$

197 $2^{x+1}+2^{3-x}=17$

$$2^x\cdot 2+2^3\cdot 2^{-x}-17=0 \qquad 2^x=t$$

$$2t+\frac{8}{t}-17=0$$

$$2t^2+8-17t=0 \qquad 2t^2-17t+8=0 \qquad \Delta=289-64=$$
$$=225=15^2$$

$$t=\frac{17\pm 15}{4}= \begin{cases} \dfrac{1}{2} \\ 8 \end{cases}$$

$2^x=\frac{1}{2} \Rightarrow x=-1$

$2^x=8 \Rightarrow x=3$

$$\boxed{x=-1 \vee x=3}$$

214 $25^{5x-2} = \sqrt[3]{125^x}$

$$(5^2)^{5x-2} = \sqrt[3]{(5^3)^x}$$

$$5^{2(5x-2)} = 5^{\frac{3x}{3}}$$

$$2(5x-2) = \frac{\cancel{3}x}{\cancel{3}}$$

$$10x - 4 = x$$

$$9x = 4 \qquad \boxed{x = \frac{4}{9}}$$

211 $4^x = 3 \cdot 2^x + 4$

$$4^x - 3 \cdot 2^x - 4 = 0$$

$$2^{2x} - 3 \cdot 2^x - 4 = 0$$

$$(2^x)^2 - 3 \cdot 2^x - 4 = 0 \qquad t = 2^x$$

$$t^2 - 3t - 4 = 0 \qquad (t-4)(t+1) = 0$$

$t = 4 \Rightarrow 2^x = 4 \quad x = 2$

$t = -1 \Rightarrow 2^x = -1$ IMPOSS.

$$\boxed{x = 2}$$

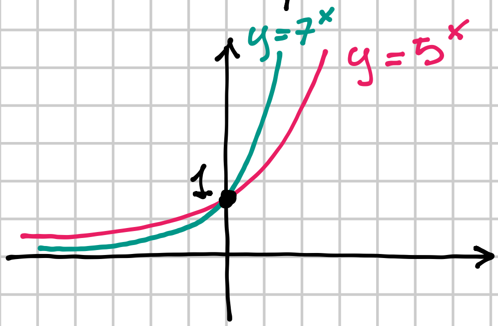
218 $5^{2x-1} = 7^{2x-1}$

$$\frac{5^{2x-1}}{7^{2x-1}} = 1$$

$$\left(\frac{5}{7}\right)^{2x-1} = 1$$

$$\left(\frac{5}{7}\right)^{2x-1} = \left(\frac{5}{7}\right)^{0} \Rightarrow 2x-1=0 \qquad \boxed{x = \frac{1}{2}}$$

Da $5^{2x-1} = 7^{2x-1}$ si può anche dedurre subito $2x-1=0$



$y=5^x$ e $y=7^x$
si intersecano solo in $(0,1)$,
cioè quando l'esponente è 0

243 $10^x - 2^x - 5^x + 1 = 0$ [0]

$$2^x \cdot 5^x - 2^x - 5^x + 1 = 0$$

$$2^x(5^x - 1) - (5^x - 1) = 0$$

$$(5^x - 1)(2^x - 1) = 0$$

$\nearrow$ $5^x - 1 = 0$ $\quad 5^x = 1$ $\quad x = 0$

$\searrow$ $2^x - 1 = 0$ $\quad 2^x = 1$ $\quad x = 0$

$$\boxed{x = 0}$$

242 $4^{\sqrt{x+2}} + 6 = 4^{2-\sqrt{x+2}}$ $\left[-\frac{7}{4}\right]$

C.E. $x+2 \geq 0 \quad x \geq -2$

$$4^{\sqrt{x+2}} + 6 = \frac{4^2}{4^{\sqrt{x+2}}} \qquad t = 4^{\sqrt{x+2}}$$

$$t + 6 = \frac{16}{t}$$

$$t^2 + 6t - 16 = 0$$

$$(t+8)(t-2) = 0$$

$t = -8$ N.A. perché $t > 0$

$t = 2 \qquad 4^{\sqrt{x+2}} = 2 \qquad 2^{2\sqrt{x+2}} = 2$

$\Rightarrow 2\sqrt{x+2} = 1 \qquad 4(x+2) = 1 \qquad x + 2 = \frac{1}{4} \qquad x = \frac{1}{4} - 2 = -\frac{7}{4}$

$$\boxed{x = -\frac{7}{4}}$$

241 $|8^x - 2| = \sqrt{2^{3x}}$ $\left[\frac{2}{3}; 0\right]$

$|x| = a \qquad a > 0$

$\Updownarrow$

$x = \pm a \iff x^2 = a^2$

1° modo

$8^x - 2 = \sqrt{2^{3x}} \quad \vee \quad 8^x - 2 = -\sqrt{2^{3x}}$

.....

2° modo

$8^{2x} + 4 - 4 \cdot 8^x = 2^{3x}$

$(8^x)^2 + 4 - 4 \cdot 8^x - (2^3)^x = 0$

$(8^x)^2 - 4 \cdot 8^x - 8^x + 4 = 0$

$(8^x)^2 - 5 \cdot 8^x + 4 = 0 \qquad 8^x = t$

$t^2 - 5t + 4 = 0 \qquad (t-4)(t-1) = 0$

$t = 4 \qquad 8^x = 4$

$t = 1 \qquad 8^x = 1$

$8^x = 4 \qquad 2^{3x} = 2^2 \Rightarrow 3x = 2 \qquad x = \frac{2}{3}$

$8^x = 1 \qquad x = 0$

$$\boxed{x = 0 \quad \vee \quad x = \frac{2}{3}}$$

239 $3^{-x} + \dfrac{3^x + 2}{3^x + 6} = \dfrac{24}{3^{2x} + 6 \cdot 3^x}$

(i denominatori sono positivi, quindi non ci sono C.E.)

$3^x = t$

$$\frac{1}{t} + \frac{t+2}{t+6} = \frac{24}{t^2 + 6t}$$

$t(t+6)$

$$\frac{t+6+t^2+2t}{\cancel{t(t+6)}} = \frac{24}{\cancel{t(t+6)}}$$

$t^2 + 3t - 18 = 0$

$(t+6)(t-3) = 0$

$t = -6$ N. ACC. (perché $t = 3^x$ non può essere negativo)

$t = 3$ $\quad 3^x = 3 \quad \boxed{x = 1}$

254

$$\begin{cases} 2^x + y = 0 \\ 4^x + y = 2 \end{cases}$$

$[(1; -2)]$

$$\begin{cases} y = -2^x \\ 4^x - 2^x = 2 \end{cases} \qquad \begin{cases} y = -2^x \\ 2^{2x} - 2^x - 2 = 0 \end{cases} \qquad 2^x = t$$

$$t^2 - t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$t = 2 \quad 2^x = 2 \quad x = 1$

$t = -1$ N.Acc.

$$\begin{cases} x = 1 \\ y = -2^x \end{cases} \qquad \begin{cases} x = 1 \\ y = -2 \end{cases} \qquad \boxed{(1, -2)}$$